Introductory Linear Algebra Analysis

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Contents

1 Executive Summary

The objective of this study was to determine the degree of commonality in Introductory Linear Algebra courses in the British Columbia transfer system. It was hoped that an agreement similar to that found in the Core Calculus Report (2002) could be forged as an aid in writing and articulating curriculum for Introductory Linear Algebra.

- The form which is appended to this document was filled in by a person from each institution who ranked each of the 93 topics as "core," "additional," or "omit."
- The raw results can be found in Section 7.
- A list of the topics together with counts of how many institutions rated them as core, additional, or omit can be found in Section 7.3.
- A description of the method based on barycentric coordinates which was used to partition the data into the three categories is found in Section 5.
- The resulting list of Core and Additional topics is found in Section 6. The topics which were omitted are found in Appendix B.

This report was written using LuaLATEX. The raw responses were entered into Lua tables and then Lua scripts were used to generate the typeset LATEX tables, do the calculations needed to generate the graph in Figure 2, do the partitioning, etc.

2 Background and Objectives

When the Core Calculus Report was completed in 2002, Introductory Linear Algebra was identified as the logical next step. After some preliminary research, it was concluded that there was too much variation between courses (particularly in their length, one or two semesters) to make agreement likely, so the idea was dropped.

After ten years some members of the BCcupms started to express the opinion that it was time to reconsider a Core Linear Algebra Study. Between the 93rd and 94th meetings (2015 and 2016) the committee chair, Jim Bailey, tried to compile a table of Linear Algebra courses together with topics which were taught in each. The difficulty of finding and interpreting information on the internet made this approach unsatisfactory, and he decided that collecting the information by having a representative from each fill in a form would be easier and more accurate. To this end he drafted a fillable pdf form based on the table of contents of the Creative Commons (CC BY) text by Ken Kuttler (distributed and supported by Lyryx Learning). This text was chosen because it is freely available and could be consulted in case there were any questions about the topics on the form. This preliminary version of the form was presented at the 94th meeting; it was agreed that this approach was worth pursuing, and a committee consisting of Nora Franzova, Claude Laflamme, Michael Nyenhuis, Wesley Snider, and Jim Bailey was struck.

3 Activities of the Committee

Our committee analysed and modified the form. Some topics were added while others were removed. We also considered how best to obtain the information which we wanted:

- what scale should be used (3 or 5 point);
- what sort of question should be asked to encourage the intended frame of mind in the respondant:
  - How often do you teach the following topics (always, sometimes, never)?
  - Where should the following topics be placed (core, additional, omitted)?
  - how important are the following topics (critical, very important, important, somewhat important, up to the instructor's discretion)?

The form originally contained many more topics. There was a fear that its length might discourage some people from filling it out, so topics which might be considered as "applications" were omitted on the theory that they would be taught as examples of the more general theory.

When the form was deemed satisfactory, a representative from each institution completed the form and submitted it to Jim Bailey. As the results came in, Jim entered the data into Lua tables as part of a LaTeX document in preparation for analysis (see Section 7). The original intention was to sum each of the columns and use that to partition the topics into Core, Additional and Omit, but, after a little reflection, Jim decided that it would be more flexible to use barycentric coordinates as described in Section 5.

Jim wrote the Lua scripts which did the analysis; updated results were available as soon as new data were in.

4 Recommendations

Notes: There is a disparity between one- and two-semester courses, but not to the extent which we expected. In particular, four universities leave the following topics to their second course:

University of Northern British Columbia:

- Complex Numbers;
- Complex Eigenvalues; Applications of Spectral Theory; Orthogonality;
- General Vector Spaces;
- Isomorphisms, Homomorphisms, Changing Map Representations; and
- Inner Product Spaces, Proofs; Axioms, Length, Angle and Orthogonality, Least Squares, L^2-Spaces.

University of Alberta:

University of Calgary:

- Vector spaces, subspaces, independence, basis and dimension, row and column space of a matrix, rank, applications.
- Linear transformations, kernel and image, composition, linear functionals, the double dual, transpose of a linear transformation.

- Orthogonality, Gram-Schmidt process, orthogonal diagonalization and least squares approximation, quadratic forms, singular value decomposition.
- Change of basis

University of Lethbridge:

This study focuses on introductory courses in linear algebra, rather than a first course in linear algebra (which is followed by a second course).

In addition, the institutes of technology (SAIT, NAIT, BCIT, and NVIT) have linear algebra courses with different content depending on the area of study (electronics, mining, forestry, mechatronics, etc.); this specialised material is considered to be essential to the respective programs. This study does not consider these courses.

The main objectives of this agreement are

- to aid sending institutions to design introductory linear algebra courses which they can be reasonably certain will transfer to other institutions;
- to guide receiving institutions in assessing the suitability of introductory linear algebra courses which have been proposed for transfer; and
- to provide guidelines for deleting topics from courses which have become overloaded. Courses tend to grow by aggregation of desirable topics until it becomes impossible to teach all of them effectively. If there are 60 topics which must be covered in a 15-week semester, then we must average 4 topics each week. This is manageable when there are 4 hours per week but is a stretch when there are only 3. The plurality partition has 67 core topics which suggests that the Introductory Linear Algebra courses at many institutions have more topics than can reasonably be taught.

Recommendations:

1. that the BCcupms accept this report and endorse the curricula as described in this document;

2. that receiving institutions grant transfer credit for Introductory Linear Algebra courses from other British Columbia post-secondary institutions whose courses are consistent with the curricula as described in this report;

3. that, when designing or modifying introductory linear algebra courses, British Columbia post-secondary mathematics departments strive to include the topics as described in this report;

4. that any post-secondary institution which has concerns about the introductory linear algebra curricula as described in this report contact the chair and have them put on the agenda for discussion at the next regularly scheduled meeting of the BCcupms; and

5. that the Introductory Linear Algebra curricula be subject to a full review at least once every five years.

5 Method of Analysis

Responses from each of the universities and colleges are given in Section 7 and summary data is given in Section 7.3.

If we let x be the proportion of the respondents who indicated 'core,' y the proportion of the respondents who indicated 'additional,' and z the proportion of the respondents who indicated 'omit,' then the point (x, y, z) satisfies

$$x + y + z = 1,\ x \geq 0,\ y \geq 0,\ \text{and}\ z \geq 0$$

which is a simplex in the first octant. By choosing a point (a, b, c) in this simplex we divide it into three regions kite-shaped regions based on barycentric coordinates. See Figure 1.

The point (x_0, y_0, z_0) is in the region containing

- the x-vertex if and only if

$$x_0 > \frac{a}{b} y_0 \text{ and } x_0 > \frac{a}{c} z_0;$$

- the y-vertex if and only if

$$y_0 > \frac{b}{c} z_0 \text{ and } y_0 > \frac{b}{a} x_0;$$

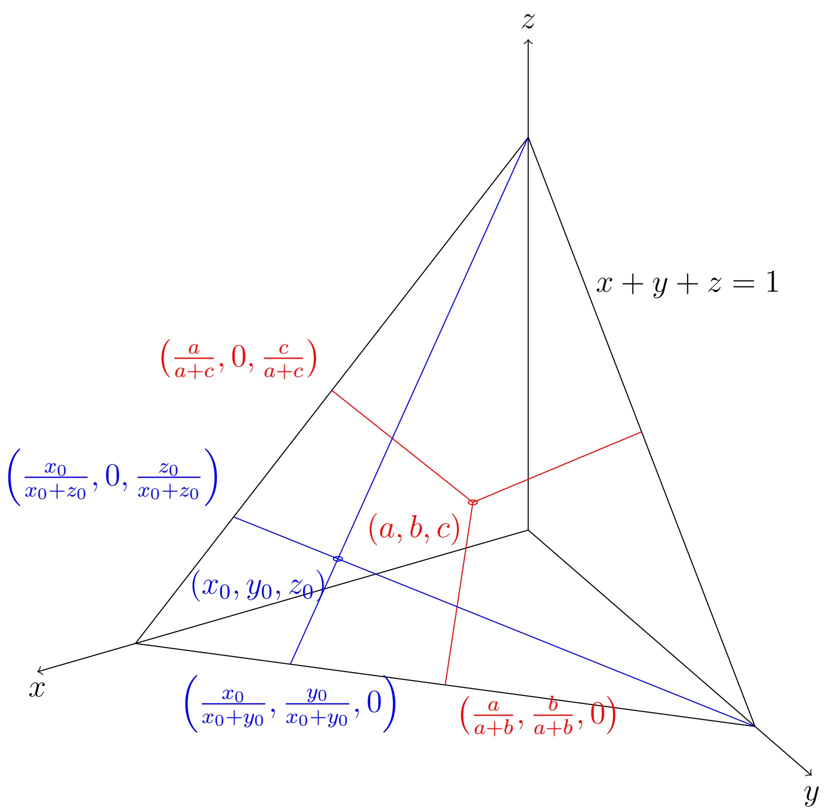

Figure 1: Partition the Response Simplex Based on Barycentric Coordinates.

- the z-vertex if and only if

$$z_0 > \frac{c}{a}x_0 \text{ and } z_0 > \frac{c}{b}y_0.$$

There is considerable agreement about which topics should be in a first course in Linear Algebra. To illustrate this, $(a, b, c) = \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$ corresponds to taking the category with the most votes (plurality). This results in

$$\text{Core} : \text{Additional} : \text{Omit} = 68 : 8 : 17$$

Alternatively, we have 93 topics which we wish to partition into three categories. Taking the Core Calculus Report as a model, we expect that Core topics will take 75% of the time and Additional topics 25% of the time (3 parts to 1). If we choose approximately one half of the Additional topics and omit an equal number, then we want

$$\text{Core} : \text{Additional} : \text{Omit} = 3 : 2 : 1 = 45 : 30 : 15$$

with 3 topics left over.

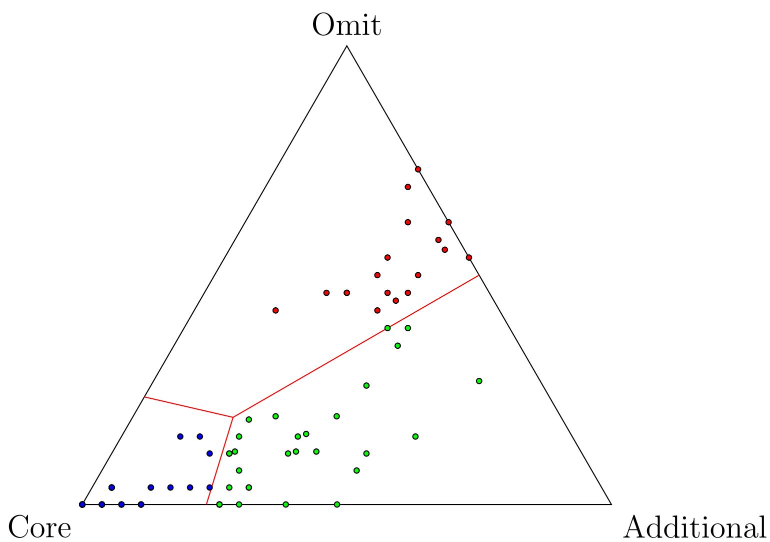

Figure 2: Barycentric Partition of the Responses.

If we take $(a, b, c) = (0.62, 0.19, 0.19)$, then

$$\text{Core} : \text{Additional} : \text{Omit} = 45 : 30 : 18$$

which is close. The 17 topics which were omitted in the plurality partition are still omitted, together with 6.1 Complex Numbers. The rest of this report is based on this partition.

6 Science Linear Algebra: Core and Additional Topics

A first year (one semester) Linear Algebra Course for Sciences must include all 45 topics from the Core Topics list. Coverage of this material should constitute approximately 75% of the course and the remaining 25% should be topics chosen from the Additional Topics list. For breadth, at least 15 Additional Topics should be included, for a total of 60 topics.

Core Topics:

Systems of Equations

1.1 Systems of Equations, Geometry
1.2 Systems of Equations, Algebraic Procedures
1.2.1 Elementary Operations
1.2.2 Gaussian Elimination
1.2.4 Rank and Homogeneous Systems

Matrices

2.1 Matrix Arithmetic
2.1.1 Addition of Matrices
2.1.2 Scalar Multiplication of Matrices
2.1.3 Multiplication of Matrices
2.1.5 Properties of Matrix Multiplication
2.1.6 The Transpose
2.1.7 The Identity and Inverses
2.1.8 Finding the Inverse of a Matrix
2.1.10 Properties of Invertible Matrices

Determinants

3.1 Basic Techniques and Properties

Inner Product Spaces, Proofs

Students must write simple proofs throughout the course

Addditional Topics:

Systems of Equations

1.2.3 Uniqueness of Reduced Row-Echelon Form

Matrices

2.1.9 Elementary Matrices

Determinants

3.2 Applications of the Determinant
3.2.1 A Formula for the Inverse
3.2.2 Cramer's Rule

$\mathbb{R}^n$

4.6 Parametric Lines
4.7.2 The Geometric Significance of the Dot Product
4.7.3 Projections
4.8 Planes in $\mathbb{R}^n$
4.9 The Cross Product
4.11 Orthogonality and the Gram Schmidt Process
4.11.1 Orthogonal and Orthonormal Sets
4.11.2 Orthogonal Matrices
4.11.3 Gram-Schmidt Process
4.11.4 Orthogonal Projections
4.11.5 Least Squares Approximation

Linear Transformations

5.4 Special Linear Transformations in the Euclidean Plane
5.5 Linear Transformations which are One-to-One or Onto
5.6 Isomorphisms
5.7 The Kernel And Image Of A Linear Map
Changes of Basis in $\mathbb{R}^n$

Complex Numbers

6.4	The Quadratic Formula

Spectral Theory

7.1.3	Eigenvalues and Eigenvectors for Diagonal Matrices
	Eigenvalues and Eigenvectors for Similar Matrices
7.2.2	Complex Eigenvalues
7.3	Applications of Spectral Theory

Vector Spaces

9.1	Algebraic Considerations
	Range and Null Spaces
	Change of Basis
	Changing Representations of Vectors

7 Institutional Responses

The following institutions are missing from this analysis:

Nicola Valley Institute of Technology: Al Fukushima informed us that NVIT does not offer a core Linear Algebra course.

Northwest Community College: Erfan S. Zahra'i has informed us that their Linear Algebra course is MATH 235 but it has never been offered.

Quest University: Glen Van Brummelen emailed us saying: "Given the unique nature of our institution as a centre for pedagogical experimentation, we don't feel it's appropriate for us to participate in this process."

In the following tables the three point scale has been replaced by numerical values:

Core: 2; Additional: 1; Omit: 0.

7.1 Linear Algebra for Science

Systems of Equations

Institution	ID	hours	1.1	1.2	1.2.1	1.2.2	1.2.3	1.2.4
SFU	Math 240	3	2	2	2	2	1	2
UBCV	Math 221	3	2	2	2	2	1	2
UNBC	MATH 220	3	2	2	2	2	2	2
UVic	Math 211	3	2	2	2	2	1	2
UAlb	Math 125	3		2	2	2	1	2
UCal	Math 211	3	2	2	2	2	1	2
ULeth	Math 1410	4	2	2	2	2	0	2
CapU	Math 200	3+1	2	2	2	2	2	2
KPU	MATH 2232	4	2	2	2	2	2	2
TRU	MATH 2120	3	2	2	2	2	2	2
TRUO	Math 2121	3	2	2	2	2	2	2
TWU								
UBCO	Math 221	3	2	2	2	2	1	2
UFV	Math 221	4	2	2	2	2	2	2
VIU	Math 241	4	2	2	2	2	2	1
Alex	Math 232	4	2	2	2	2	2	1
Cam	Math 125	5	2	2	2	2	2	2
CNC	Math 204	3	2	2	2	2	2	2
CotR	MATH 221	3	2	2	2	2	1	2
ColC	MATH 252	4	2	2	2	2	2	2
CoqC	Math 232	3	2	2	2	2	2	2
Doug	MATH 2232	4	2	2	2	2	2	2
Lang	Math 2362	4	2	2	2	2	2	2
NIC	MAT 200	4.5	2	2	2	2	2	2
NLC	Math 152	4	2	2	2	2	1	1
OC	Math 221	4	2	2	2	2	2	2
Selk	MATH 221	5	2	2	2	2	2	2
VCC	MATH 1221	4	2	2	2	2	2	2
YC								
BCIT								
Core:			26	27	27	27	18	24
Additional:			0	0	0	0	8	3
Omit:			0	0	0	0	1	0

Matrices

	2.1	2.1.1	2.1.2	2.1.3	2.1.5	2.1.6	2.1.7	2.1.8	2.1.9	2.1.10
SFU	2	2	2	2	2	2	2	2	2	2
UBCV	2	2	2	2	2	2	2	2	1	2
UNBC	2	2	2	2	2	2	2	2	2	2
UVic	2	2	2	2	2	2	2	2	2	2
UAlb	2	2	2	2	2	2	2	2	2	2
UCal	2	2	2	2	2	2	2	2	2	2
ULeth	2	2	2	2	2	2	2	2	1	2
CapU	2	2	2	2	2	1	2	2	1	2
KPU	2	2	2	2	2	2	2	2	2	2
TRU	2	2	2	2	2	2	2	2	1	2
TRUO	2	2	2	2	2	2	2	2	1	2
TWU										
UBCO	2	2	2	2	2	2	2	2	1	2
UFV	2	2	2	2	2	2	2	2	2	2
VIU	2	2	2	2	2	2	2	2	1	2
Alex	2	2	2	2	2	2	2	2	2	2
Cam	2	2	2	2	2	2	2	2	2	2
CNC	2	2	2	2	2	2	2	2	1	2
CotR	2	2	2	2	2	2	2	2	2	2
ColC	2	2	2	2	2	2	2	2	2	2
CoqC	2	2	2	2	2	2	2	2	2	2
Doug	2	2	2	2	2	2	2	2	2	2
Lang	2	2	2	2	2	2	2	2	1	2
NIC	2	2	2	2	2	2	2	2	1	2
NLC	2	2	2	2	2	2	2	2	1	1
OC	2	2	2	2	2	1	2	2	1	2
Selk	2	2	2	2	2	2	2	2	1	2
VCC	2	2	2	2	2	2	2	2	2	1
YC										
BCIT										
Core:	27	27	27	27	27	25	27	27	14	25
Additional:	0	0	0	0	0	2	0	0	13	2
Omit:	0	0	0	0	0	0	0	0	0	0

Determinants

	3.1	3.1.1	Lapl	3.1.2	3.1.3	Perm	Form	3.1.4	3.2	3.2.1	3.2.2
SFU	2	2	2	2	2	0	0	1	2	2	2
UBCV	2	2	2	2	2	1	1	2	1	1	1
UNBC	2	2	2	2	2	0	2	2	2	2	2
UVic	2	2	2	2	2	0	0	2	2	2	2
UAlb	2	2	2	2	2	0	0	2	1	2	1
UCal	2	2	2	2	2	2	0	1	2	2	2
ULeth	2	2	2	2	2	0	0	2	2	1	1
CapU	2	2	2	2	2	1	1	2	1	1	1
KPU	2	2	2	1	2	1	1	2	1	1	0
TRU	2	2	2	2	2	0		1	2	2	1
TRUO	2	2	2	2	2	1	0	2	2	2	1
TWU											
UBCO	2	2	1	2	2	1	0	2	1	0	0
UFV	2	2	2	2	2	0	0	2	2	0	0
VIU	2	2	2	2	2	0	0	2	1	1	2
Alex	2	2	2	2	2	0	0	2	2	1	1
Cam	2	2	2	2	2	0	0	2	2	1	1
CNC	2	2	2	2	2	1	0	2	2	2	1
CotR	2	2	2	2	2	1	1	2	2	2	2
ColC	2	2	2	2	2	1	1	2	2	2	2
CoqC	2	2	2	2	2	1	1	2	2	2	2
Doug	2	2	2	2	2	1	1	2	2	2	2
Lang	2	2	2	1	2	1	0	1	2	1	1
NIC	2	2	2	2	2	0	0	1	2	1	2
NLC	2	2	2	1	1	0	0	1	2	1	2
OC	2	2	2	2	2	0	0	2	2	1	1
Selk	2	2	2	2	2	0	0	2	2	1	1
VCC	2	2	2	2	2	0	0	0	1	1	1
YC											
BCIT											
Core:	27	27	26	24	26	1	1	20	20	12	11
Additional:	0	0	1	3	1	11	7	6	7	13	13
Omit:	0	0	0	0	0	15	18	1	0	2	3

$\mathbb{R}^n$

	4.1	4.2	4.2.1	4.2.2	4.3	4.4	4.5	4.6	4.7.1	4.7.2	4.7.3	4.8	4.9	4.9.1
SFU	2	2	2	2	2	2	2	1	2	1	2	1	0	0
UBCV	2	2	2	2	2	2	1	1	2	2	2	2	1	0
UNBC	2	2	2	2	2	2	2	2	2	2	1	2	2	0
UVic	2	2	2	2	2	2	2	2	2	2	2	2	2	1
UAlb	2	2	2	2	2	2	2	2	2	1	1	2	0	0
UCal	2	2	2	2	2	2	2	2	2	2	2	2	2	2
ULeth	2	2	2	2	2	2	2	2	2	2	2	2	2	1
CapU	2	2	2	2	1	1	1	1	2	1	1	1	1	1
KPU	2	2	2	2	2	2	2	1	2	2	2	2	1	1
TRU	2	2	2	2	2	2	2	2	2	2	2	2	2	0
TRUO	2	2	2	2	2	2	2	2	2	2	2	2	2	1
TWU														
UBCO	2	2	2	2	2	1	2	0	2	1	0	2	0	0
UFV	2	2	2	2	2	2	2	1	2	2	2	1	0	0
VIU	2	2	2	2	2	2	2	1	2	2	2	1	1	0
Alex	2	2	2	2	2	2	2	2	2	1	2	2	1	0
Cam	2	2	2	2	2	2	2	2	2	2	2	2	2	2
CNC	2	2	2	2	2	2	2	2	2	2	1	1	1	0
CotR	2	2	2	2	2	2	2	2	2	2	2	2	2	2
ColC	2	2	2	2	2	2	2	2	2	2	2	2	2	2
CoqC	2	2	2	2	2	2	2	2	2	2	2	2	2	1
Doug	2	2	2	2	2	2	2	2	2	2	2	1	1	0
Lang	2	2	2	2	2	2	2	1	1	1	1	1	1	0
NIC	2	2	2	2	2	2	2	2	2	2	2	2	2	1
NLC	2	2	2	2	2	2	2	2	2	2	1	2	2	1
OC	2	2	2	2	1	1	1	1	2	1	1	1	1	1
Selk	2	2	2	2	2	2	2	2	2	2	2	2	2	2
VCC	2	2	2	2	2	2	2	2	2	2	2	2	2	1
YC														
BCIT														
Core:	27	27	27	27	25	24	24	18	26	20	19	19	14	5
Additional:	0	0	0	0	2	3	3	8	1	7	7	8	9	10
Omit:	0	0	0	0	0	0	0	1	0	0	1	0	4	12

$\mathbb{R}^n$

	4.10	4.10.1	4.10.2	4.10.4	4.10.5	4.11	4.11.1	4.11.2	4.11.3	4.11.4	4.11.5
SFU	2	2	2	2	2	2	2	1	2	2	2
UBCV	2	2	2	2	2	1	1	1	1	1	1
UNBC	2	2	2	2	2	0		2			
UVic	2	2	2	2	2	2	2	2	2	2	1
UAlb	2	2	2	2	2	0	0	0	0	0	0
UCal	0	0	0	0	0	0	0	0	0	0	0
ULeth	1	1	1	1	1	1	1	1	1	1	1
CapU	2	2	2	2	1	1	1	1	1	1	0
KPU	2	2	2	2	2	2	2	1	2	2	1
TRU	2	2	2	2	1	2	2	0	1	2	1
TRUO	2	2	2	2	2	2	2	1	2	2	0
TWU											
UBCO	2	2	2	2	2	0	0	0	0	0	0
UFV	2	2	2	2	2	2	2	1	2	2	1
VIU	2	2	2	2	2	1	2	1	1	1	1
Alex	2	2	2	2	2	2	2	2	2	2	2
Cam	2	2	2	2	2	2	2	2	2	2	0
CNC	2	2	2	2	2	2	1	1	1	1	1
CotR	2	2	2	2	2	2	2	2	1	2	1
ColC	2	2	2	2	2	2	2	1	2	2	1
CoqC	2	2	2	2	2	2	2	2	2	2	1
Doug	2	2	2	2	2	2	2	2	2	2	1
Lang	2	2	2	2	2	2	2	1	2	1	1
NIC	2	2	2	2	2	1	1	1	1	1	0
NLC	2	2	2	2	1	2	2	1	1	1	1
OC	2	2	2	2	2	1	1	1	1	1	1
Selk	2	2	2	2	2	2	2	1	2	2	1
VCC	2	2	2	2	2	2	2	2	2	2	2
YC											
BCIT											
Core:	25	25	25	25	22	17	17	8	13	14	3
Additional:	1	1	1	1	4	6	6	15	10	9	16
Omit:	1	1	1	1	1	4	3	4	3	3	7

Linear Transformations; Complex Numbers

	5.1	5.2	5.3	5.4	5.5	5.6	5.7	5.9	Basis	6.1	6.2	6.3	6.4
SFU	2	2	2	2	2	1	2	2	2	2	2	0	2
UBCV	2	2	2	2	1	1	2	2	1	1	1	1	1
UNBC	0	0	0	0	0	0	2	2					
UVic	2	2	2	2	2	0	2	2	1	0	0	0	0
UAlb	2	2	2	1	2	2	2	2	0	0	0	0	0
UCal	2	2	2	2	1	1	0	0	0	2	2	2	2
ULeth	1	1	1	0	0	0	1	0	0	2	1	1	1
CapU	2	2	2	1	1	1	1	2	2	1	1	0	1
KPU	2	2	2	0	2	0	2	2	0	0	0	0	0
TRU	2	2	2	2	1	1	1	1	2	2	2	1	2
TRUO	2	2	2	1	2	2	2	2	2	0	0	0	1
TWU													
UBCO	2	2	2	1	2	0	1	2	1	0	0	0	0
UFV	2	2	2	2	1	0	1	2	2	0	0	0	0
VIU	2	2	2	2	2	2	2	2	2	1	1	1	1
Alex	2	2	2	2	2	1	2	1	2	2	1	1	1
Cam	2	2	2	2	2	2	2	2	2	2	2	2	2
CNC	2	2	2	1	1	1	1	2	1	0	0	0	0
CotR	2	2	2	1	2	2	2	2	2	2	2	1	1
ColC	2	2	2	2	0	0	0	0	2	0	0	0	0
CoqC	2	2	2	2	2	1	2	2	1	2	2	2	2
Doug	2	2	2	1	2	2	2	1	2	0	0	0	0
Lang	2	2	2	1	2	2	2	1	1	0	0	0	0
NIC	2	2	2	2	2	1	1	1	1	0	0	0	0
NLC	2	2	2	2	2	1	2	2	1	2	0	0	1
OC	2	2	2	2	2	1	2	2	2	1	1	1	1
Selk	2	2	2	1	2	2	2	2	1	2	2	1	1
VCC	2	2	2	2	2	2	2	2	2	2	2	2	2
YC													
BCIT													
Core:	25	25	25	15	18	9	18	19	13	11	8	4	6
Additional:	1	1	1	9	6	11	7	5	9	4	6	8	10
Omit:	1	1	1	3	3	7	2	3	4	11	12	14	10

Spectral Theory

	7.1	7.1.1	7.1.2	7.1.3	Sim	7.2	7.2.1	7.2.2	7.3	7.4	7.4.1	7.4.2	7.4.3	7.4.4
SFU	2	2	2	2	2	2	2	2	0	0	0	0	1	0
UBCV	2	2	2	2	1	2	2	1	1	1	1	1	1	1
UNBC	2	2	2	2	2	2	2							
UVic	2	2	2	2	2	2	2	0	1	1	1	1	0	1
UAlb	2	2	2	2	2	2	2	0	0	0	0	0	0	0
UCal	2	2	2	2	2	2	2	2	2	0	0	0	0	0
ULeth	2	2	2	2	2	1	1	1	1	1	1	0	0	0
CapU	2	2	2	2	2	2	2	0	1	0	0	0	0	0
KPU	2	2	2	2	2	0	0	0	0	0	0	0	0	0
TRU	2	2	2	2	1	2	2	2	1	1	0	1	1	0
TRUO	2	2	2	2	2	2	2	0	2	0	0	0	0	0
TWU														
UBCO	2	2	2	1	1	2	2	1	0	0	0	0	0	0
UFV	2	2	2	2	2	2	2	0	2	0	0	0	0	0
VIU	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Alex	2	2	2	2		2	2	2	1	2	2	0	1	1
Cam	2	2	2	2	2	2	2	0	0	2	2	0	0	0
CNC	2	2	2	1	1	1	1	0	0	0	0	0	0	0
CotR	2	2	2	1	2	2	2	1	2	2	2	2	1	1
ColC	2	2	2	2	2	2	2	0	0	0	0	0	0	0
CoqC	2	2	2	2	2	2	2	1	2	2	2	1	1	1
Doug	2	2	2	2	2	2	2	0	1	0	0	0	1	0
Lang	2	2	2	1	2	2	2	1	0	1	1	0	0	1
NIC	2	2	2	2	1	2	2	1	0	0	0	0	0	0
NLC	2	2	2	1	1	2	2	1	1	1	1	1	1	1
OC	2	2	2	1	1	2	2	1	1	1	1	1	1	0
Selk	2	2	2	2	1	2	2	1	1	1	1	1	1	1
VCC	2	2	2	2	1	2	2	2	2	2	2	2	1	1
YC														
BCIT														
Core:	26	26	26	20	16	23	23	5	6	5	5	2	0	0
Additional:	1	1	1	7	10	3	3	11	11	9	8	8	12	10
Omit:	0	0	0	0	0	1	1	10	9	12	13	16	14	16

Vector Spaces

	9.1	9.2	9.3	Iso	Homo	Range	ChngBas	ChngRep	ChngMap
SFU	2	2	2	2	1	2	2	2	1
UBCV	1	1	1	0	0	1	1	1	0
UNBC	2	2	2			2			
UVic	2	2	2	1	1	2	1	2	0
UAlb	0	0	0	0	0	0	0	0	0
UCal	0	0	0	0	0	0	0	0	0
ULeth	1	1	1	0	0	0	0	0	0
CapU	2	2	2	0	0	2	2	1	1
KPU	2	2	2	0	0	2	2	2	2
TRU	2	2	2	1	1	1	1	1	1
TRUO	2	2	2	0	0	2	2	2	1
TWU									
UBCO	1	0	0	0	0	0	0	0	0
UFV	2	2	2	1	2	1	2	2	2
VIU	2	2	2	2	1	2	2	2	1
Alex	2	2	2	1	1	2	2	2	0
Cam	2	2	2	2	0	2	2	2	0
CNC	2	2	2	1	1	1	1	1	1
CotR	2	2	2	2	2	2	2	2	2
ColC	2	2	2	0	0	2	2	2	0
CoqC	0	0	0	0	0	0	0	0	0
Doug	2	2	2	2	0	2	2	2	2
Lang	1	1	2	1	1	2	2	1	0
NIC	2	2	2	0	0	2	1	1	0
NLC	1	1	1	1	1	1	1	1	1
OC	2	2	2	1	1	2	2	1	1
Selk	2	2	2	2	2	2	1	1	1
VCC	1	2	2	1	1	1	2	1	1
YC									
BCIT									
Core:	18	19	20	6	3	16	14	11	4
Additional:	6	4	3	9	10	6	7	10	10
Omit:	3	4	4	11	13	5	5	5	12

Inner Product Spaces; Introduction to Proofs; Miscellaneous

	Axiom	Len	LstSq	L^2	Proofs	PreReq	Soft	Soft	Compulsory?
SFU	1	1	0	0	2	Calc 1	N	MATLAB	N
UBCV	0	0	0	0	1	Calc 2	R	MATLAB	Y
UNBC					2	Calc 1	N	Other	
UVic	0	0	0	0	2	Calc 1	N	None	
UAlb	0	0	0	0	2	None	N	None	
UCal	0	0	0	0	0	None	E	Other	
ULeth	0	0	0	0	2	None	E	GeoGebra	Y
CapU	0	0	0	0	2	Calc 2	R	TI-89	
KPU	0	0	0	0	2	Calc 1		Maple	
TRU	1	1	1	1	2	6 Math credits	E	Wolfram Alpha	
TRUO	2	2	0	0	2	Calc 2	N	None	
TWU									
UBCO	0	0	0	0	2	Calc 1	R	Maple	Y
UFV	2	2	1	0	2	Calc 2	R	Maple	
VIU	2	2	1	0	2	Calc 2	N	Maple	
Alex	2	2	2	0	2	Calc 1	N	None	
Cam	2	2	0	0	2	Calc 1	N	None	
CNC	1	1	1	1	1	None	N		
CotR	2	2	1	1	2	Calc 1	E	Maple	
ColC	0	0	0	0	2	Calc 2	N		
CoqC	0	0	0	0	2	Calc 1	N	None	
Doug	2	2	1	0	2	Calc 1	E	Maple	Y
Lang	0	0	0	0	1	Calc 2	R	MATLAB	
NIC	0	0	0	0	2	MAT 1xx	R	Geogebra	
NLC	1	1	1	1	2	Calc 1	N		Y
OC	1	1	1	1	1	Calc 1	R	Maple	Y
Selk	1	1	1	1	2	Calc 1	N	None	Y
VCC	1	1	1	1	1	Calc 1	E	MATLAB	
YC									
BCIT									
Core:	7	7	1	0	21				
Additional:	7	7	10	7	5				
Omit:	12	12	15	19	1				

7.1.1 Notes:

- Alexander College: J. Lennart Berggren (submitted by Krishna Subedi).
  Text: Linear Algebra and its Applications (5th ed.) by Lay, David C., Lay, Steven R., McDonald, Judi J.

- Camosun College: Dan Bergerud.
  Text: Elementary Linear Algebra by Larson.
  I have just changed to the Larson, Elementary Linear Algebra text and will do these sections:

  - Systems of Linear Equations 1.1 – 1.2
  - Matrices 2.1 – 2.4
  - Determinants 3.1 – 3.3
  - Vector Spaces 4.1 – 4.7
  - Inner Product Spaces 5.1 – 5.3

- Capilano University: Lisa Lajeunesse/Chris Morgan
  Text: Linear Algebra: A Modern Introduction by David Poole
  3 hours in class plus 1 additional hour delivered through online exercises.
  Our students use the matrix capabilities of TI-89. No other software is required.

- College of the Rockies: Jim Bailey.
  Text: A First Course in Linear Algebra by Ken Kuttler (open source)

- Columbia College: Ana Culibrk.
  Text: Elementary Linear Algebra by Howard Anton.
  Additional topic: an application of diagonalization, solving systems of first order differential equations.

- College of New Caledonia: Nicholas Buck
  Text: Linear Algebra by Poole.
  With three hours per week for 12 weeks, it is difficult to cover even the material we have indicated as core, let alone the additional topics.
  It is natural to follow the selected text as closely as possible, so the material for the course in a particular term could be predetermined to some extent.

- Coquitlam College: Gera Belchev.
  Text: Linear Algebra and its Applications, 4th ed. by David C.Lay.
- Douglas College: Natasha Davidson (submitted by Wesley Snider).
  Text: Linear Algebra and its Applications by David C. Lay.
  Prerequisites: Discrete Math 1 OR Calculus 1.
  Software is up to the discretion of the instructor.
- Kwantlen Polytechnic University: Michael Nyenhuis.
  Text: Elementary Linear Algebra by Anton.
- Langara: Nora Franzova.
  Text: Elementary Linear Algebra Edition 11 by Anton.
- North Island College: Dennis Lightfoot.
  Text: Introduction to Linear Algebra for Science and Engineering by Norman and Wolczuk.
- Northern Lights College: Hongbin Cui.
  Text: Elementary Linear Algebra by Howard Anton.
  Math 152 is not currently offered at NLC.
- Okanagan College: Joe Hobart.
  Text: Linear Algebra by Lay.
- Selkirk: Ryan Bradshaw.
  Text: Linear Algebra with Applications, Seventh Edition, Keith Nicholson.
- Simon Fraser University: Justin Gray.
  Text: Linear Algebra and its Applications by David C. Lay, Steven R. Lay and Judi J. McDonald.
  This course is entitled "Algebra I: Linear Algebra."
- Thompson Rivers University: Richard Taylor.
  Text: Linear Algebra: A Modern Introduction by David Poole.
  - applications of linear systems (e.g. flow networks, resource allocation)
  - applications of matrix multiplication (e.g. Markov chains, counting in graphs and networks)

- Thompson Rivers University—Open Learning: Chris Morgan.
  Text: Elementary Linear Algebra (7th ed) by Ron Larson.

- University of Alberta: David McNeilly.
  Text: Linear Algebra: A Modern Introduction by David Poole.
  The first linear algebra course at the University of Alberta, vector spaces are restricted to $\mathbb{R}^n$. Abstract vector spaces (as I assumed are covered in section 9.1–9.3 of Kuttler) are deferred to the second course. Linear transformations are restricted to those between Euclidean spaces, with the emphasis on the connection with matrix transformation and simple geomtric transformations (rotations, reflections, projection). The dot product and length are developed for $\mathbb{R}^n$, with the discussion of lines and planes generally...

- University of British Columbia. Wayne Nagata.
  Text: Linear Algebra and its Applications (UBC Edition) by David C. Lay.

- University of British Columbia (Okanagan). Wayne Broughton.
  Text: Linear Algebra with Applications, 2nd ed. by Jeffrey Holt.
  The math department at UBC Okanagan is considering possible changes to the structure of the linear algebra courses offered, to be better able to meet the needs of the students from many different programs taking these courses. So this form describes only what is currently offered in our first linear algebra course. It does not necessarily reflect a consensus view on what this course should contain, since that view could evolve.

- University of Calgary: Claude Laflamme.
  Text: A First Course in Linear Algebra by K. Kuttler, open text adapted by Lyryx Learning.
  Online homework.

- University of the Fraser Valley: Greg Schlitt (submitted by Ian Affleck).
  Text: Linear Algebra with Applications by Leon.

- University of Lethbridge: Sean Fitzpatrick.
  Custom by Hartman; Hartman et al; Stitz and Zeager; Fitzpatrick.
  At ULeth there is probably not "consensus opinion" on the topics to

be included. (I am also one of the few who makes tutorials mandatory.) When the course is taught by Dr. Kharaghani, for example, the primary focus of the course is orthogonal diagonalization, whereas I consider this to be a topic better-suited for a second course in linear algebra.

My current textbook is one that I prepared over the summer using open source materials. The primary source textbooks were:

- Precalculus, by Stitz and Zeager, for the material on complex numbers, including polar forms, powers and roots, etc.
- APEX Calculus, by Hartman et al, for the chapter on vector geometry (lines, planes, projections, etc.)
- Matrix Algebra, by Gregory Hartman, for the core linear algebra material.
- Greg's Matrix Algebra book was lacking in theory and proofs, so I wrote this in myself where required. I wrote a couple of portions myself, including the section on the vector space $\mathbb{R}^n$ (including span, linear independence, etc).
- I've found that the inclusion of vector geometry in a linear algebra course is a Western Canadian peculiarity; most other institutions where I've taught include this as part of Calculus III.
- If you want access to the book, it's available at the following URL: `http://www.cs.uleth.ca/~fitzpat/Textbooks/Texts.html` (See the section for Math 1410. There is an e-book, in colour, that has interactive 3D graphics in the vector geometry chapter, and a black and white version for printing.) The source code is published on GitHub: `https://github.com/ULeth-Math-CS/Math1410-Text`

• University of Northern British Columbia: form filled in by Iliya Bluskov, Sam Walters, and Edward Dobrowolski and submitted by Erin Beveridge.
Text: Linear Algebra with Applications by Nicholson.
Topics which are unchecked are covered in their Linear Algebra 2 (MATH 326) course. "If time or prerequisites allowed, Linear Algebra 2 would benefit greatly from complex number techniques, particularly spectral theory."

  - Inner product spaces, proofs: our students do proofs and do the dot product but not necessarily other inner product spaces.
  - Linear Transformations. The text we use has a brief section on this early on and a more complete chapter later on. The later chapter is used in our 2nd course. The earlier material is covered.

- University of Victoria: Gary MacGillivray; form answered by Marcelo Laca.
  Text: Introduction to Linear Algebra for Science and Engineering by Norman and Wolczuk.

- Vancouver Community College: Costa Karavas.
  Text: Linear Algebra and its Applications, 5th edition, by Lay et al.
  VCC has only one Linear Algebra course which is intended for First Year University Transfer, Computing Science, the Software Systems Certificate, as well as Engineering, and so has been listed under both Science and Engineering.

- Vancouver Island University: David Bigelow.
  Text: Elementary Linear Algebra by Ron Larson.
  Math 241 is our standard Linear Algebra Course, mostly taken by math and computing science students. We also offer Math 141, Matrix Algebra for Engineers, which is more computational in nature. Some of the topics we have listed as core are not always taught due to time constraints ... but we would like them to be taught.

7.2 Linear Algebra for Engineering

Systems of Equations

Institution	ID	hours	1.1	1.2	1.2.1	1.2.2	1.2.3	1.2.4
SFU	Math 232	3	2	2	2	2	1	2
UBCV	Math 152	3	2	2	2	2	1	2
UVic	Math 110	3	2	2	2	2	2	2
CapU	MATH 152	4.5+1	2	2	2	2	2	2
KPU	MATH 1152	4	2	2	2	2	0	2
UFV	MATH 152	4	2	2	2	2	0	2
Lang	Math 1252	4	2	2	2	2	1	2
NIC	MAT 133	4.5	2	2	2	2	2	2
VCC	MATH 1221	4	2	2	2	2	2	2
Core:			9	9	9	9	4	9
Additional:			0	0	0	0	3	0
Omit:			0	0	0	0	2	0
Sums:			18	18	18	18	11	18

Matrices

	2.1	2.1.1	2.1.2	2.1.3	2.1.5	2.1.6	2.1.7	2.1.8	2.1.9	2.1.10
SFU	2	2	2	2	2	2	2	2	2	2
UBCV	2	2	2	2	2	2	2	2	1	2
UVic	2	2	2	2	2	1	2	2	1	2
CapU	2	2	2	2	2	1	2	2	1	2
KPU	2	2	2	2	2	2	2	2	2	2
UFV	2	2	2	2	2	1	2	2	1	2
Lang	2	2	2	2	2	2	2	2	1	1
NIC	2	2	2	2	2	2	2	2	1	2
VCC	2	2	2	2	2	2	2	2	2	1
Core:	9	9	9	9	9	6	9	9	3	7
Additional:	0	0	0	0	0	3	0	0	6	2
Omit:	0	0	0	0	0	0	0	0	0	0

Determinants

	3.1	3.1.1	Lapl	3.1.2	3.1.3	Perm	Form	3.1.4	3.2	3.2.1	3.2.2
SFU	2	2	2	2	1	0	0	1	2	1	1
UBCV	2	2	2	2	2	2	1	2	2	1	1
UVic	2	2	2	1	1	0	0	2	2	2	2
CapU	2	2	2	2	2	0	0	2	1	1	1
KPU	2	2	2	1	2	0	0	2	0	0	0
UFV	2	2	2	1	1	0	0	1	2	1	2
Lang	2	2	2	2	2	1	0	1	2	1	1
NIC	2	2	2	2	2	0	0	1	2	1	2
VCC	2	2	2	2	2	0	0	0	1	1	1
Core:	9	9	9	6	6	1	0	4	6	1	3
Additional:	0	0	0	3	3	1	1	4	2	7	5
Omit:	0	0	0	0	0	7	8	1	1	1	1

$\mathbb{R}^n$

	4.1	4.2	4.2.1	4.2.2	4.3	4.4	4.5	4.6	4.7.1	4.7.2	4.7.3	4.8	4.9	4.9.1
SFU	2	2	2	2	2	2	2	2	2	2	2	2	0	0
UBCV	2	2	2	2	2	2	2	2	2	2	2	2	2	2
UVic	2	2	2	2	2	2	2	2	2	2	2	1	0	0
CapU	2	2	2	2	2	2	2	2	2	2	2	2	2	1
KPU	2	2	2	2	2	2	2	2	2	2	2	2	2	1
UFV	2	2	2	2	2	2	2	1	2	2	2	0	0	0
Lang	2	2	2	2	2	2	2	2	2	2	2	2	2	1
NIC	2	2	2	2	2	2	2	2	2	2	2	2	2	1
VCC	2	2	2	2	2	2	2	2	2	2	2	2	2	1
Core:	9	9	9	9	9	9	9	8	9	9	9	7	6	1
Additional:	0	0	0	0	0	0	0	1	0	0	0	1	0	5
Omit:	0	0	0	0	0	0	0	0	0	0	0	1	3	3

$\mathbb{R}^n$ cont'd

	4.10	4.10.1	4.10.2	4.10.4	4.10.5	4.11	4.11.1	4.11.2	4.11.3	4.11.4	4.11.5
SFU	2	2	2	2	2	2	2	1	2	2	1
UBCV	2	2	2	2	1	1	1	1	1	1	1
UVic	2	2	2	2	2	2	2	2	2	1	0
CapU	2	2	2	2	1	1	1	1	1	1	0
KPU	0	0	0	0	0	2	2	2	2	2	1
UFV	2	2	2	2	2	2	2	1	2	2	1
Lang	1	1	1	0	0	0	1	0	0	1	0
NIC	2	2	2	2	2	1	1	1	1	1	0
VCC	2	2	2	2	2	2	2	2	2	2	2
Core:	7	7	7	7	5	5	5	3	5	4	1
Additional:	1	1	1	0	2	3	4	5	3	5	4
Omit:	1	1	1	2	2	1	0	1	1	0	4

Linear Transformations; Complex Numbers

	5.1	5.2	5.3	5.4	5.5	5.6	5.7	5.9	Basis	6.1	6.2	6.3	6.4
SFU	2	2	2	2	2	0	2	2	2	2	2	0	2
UBCV	2	2	2	2	2	1	1	2	1	2	2	2	2
UVic	2	2	2	0	0	0	0	0	0	2	2	2	1
CapU	2	2	2	2	0	0	1	2	0	2	1	0	2
KPU	0	0	0	2	0	0	0	1	0	2	2	1	1
UFV	2	2	2	1	1	0	1	2	0	2	2	2	0
Lang	2	2	2	2	0	0	0	1	0	2	2	2	2
NIC	2	2	2	2	2	1	1	1	1	2	2	2	2
VCC	2	2	2	2	2	2	2	2	2	2	2	2	2
Core:	8	8	8	7	4	1	2	5	2	9	8	6	6
Additional:	0	0	0	1	1	2	4	3	2	0	1	1	2
Omit:	1	1	1	1	4	6	3	1	5	0	0	2	1

Spectral Theory

	7.1	7.1.1	7.1.2	7.1.3	Sim	7.2	7.2.1	7.2.2	7.3	7.4	7.4.1	7.4.2	7.4.3	7.4.4
SFU	2	2	2	2	2	2	2	2	0	0	0	0	0	0
UBCV	2	2	2	2	2	1	1	2	2	1	1	1	1	1
UVic	2	2	2	2	0	2	2	1	1	2	2	0	1	0
CapU	2	2	2	2	2	2	2	2	2	0	0	0	0	0
KPU	2	2	2	2	2	2	2	2	0	0	0	0	0	0
UFV	2	2	2	2	1	2	2	2	1	1	1	1	1	1
Lang	2	2	2	1	1	2	2	2	0	0	0	0	0	0
NIC	2	2	2	2	1	2	2	1	0	0	0	0	0	0
VCC	2	2	2	2	1	2	2	2	2	2	2	2	1	1
Core:	9	9	9	8	4	8	8	7	3	2	2	1	0	0
Additional:	0	0	0	1	4	1	1	2	2	2	2	2	4	3
Omit:	0	0	0	0	1	0	0	0	4	5	5	6	5	6

Vector Spaces

	9.1	9.2	9.3	Iso	Homo	Range	ChngBas	ChngRep	ChngMap
SFU	0	0	0	0	0	0	0	0	0
UBCV	1	1	1	1	1	1	1	2	1
UVic	0	0	0	0	0	0	0	0	0
CapU	0	0	0	0	0	0	0	0	0
KPU	0	0	0	0	0	0	0	0	0
UFV	2	2	2	0	0	0	0	0	0
Lang	1	0	1	0	0	0	2	1	0
NIC	2	2	2	0	0	2	1	1	0
VCC	1	2	2	1	1	1	2	1	1
Core:	2	3	3	0	0	1	2	1	0
Additional:	3	1	2	2	2	2	2	3	2
Omit:	4	5	4	7	7	6	5	5	7

Inner Product Spaces; Introduction to Proofs; Miscellaneous

	Axiom	Len	LstSq	L^2	Proofs	PreReq	Soft	Soft	Compulsory?
SFU	0	0	0	0	0	Calc 1	N	MATLAB	
UBCV	1	1	1	1	2	Calc 1	R	MATLAB	
UVic	0	0	0	0	1	None	R	MATLAB	
CapU	0	0	0	0	2	Calc 2	R	TI-84	
KPU	0	0	0	0	0	Calc 1xx	R	Maple	
UFV	2	2	1	2	0	Calc 1	E	Maple	
Lang	0	0	0	0	0	Calc 1	R	MATLAB	Y
NIC	0	0	0	0	1	PrCalc12	R	Geogebra	
VCC	1	1	1	1	1	Calc 1	E	MATLAB	
Core:	1	1	0	1	2				
Additional:	2	2	3	2	3				
Omit:	6	6	6	6	4				

7.2.1 Notes

- Capilano University: Lisa Lajeunesse/Marsha Anderson
  Text: Linear Algebra: A Modern Introduction (and a supplement on DEs written in-house) by David Poole
  4.5 hours of scheduled class time plus 1 hour delivered outside of class.
  Our students use the matrix capabilities of TI-84+ . No other software is required.
  Calc II is a corequisite course for Math 152.

- Kwantlen Polytechnical University: Michael Nyenhuis.
  Text: Linear Algebra, a Matrix Approach by Insel, Spence and Friedberg.

- Langara: Nora Franzova.
  Text: Introduction to Linear Algebra for Science and Engineering, Second Edition by Norman and Wolczuk.

- North Island College: Dennis Lightfoot.
  Text: Introduction to Linear Algebra for Science and Engineering by Norman and Wolczuk.

- Simon Fraser University: Justin Gray.
  Text: Contemporary Linear Algebra by Howard Anton and Robert C.

Busby.
This course is entitled "Applied Linear Algebra." It emphasizes applications, for example Markov Chains and the PageRank algorithm.

- University of British Columbia Vancouver: (Wayne Nagata).
  Text: Online Notes by Brian Wetton and Richard Froese.

- University of the Fraser Valley: Ben Vanderlei (submitted by Ian Affleck).
  Text: Linear Algebra and its Applications by Lay, Lay, and McDonald.

- University of Victoria: Christopher Eagle.
  Text: Linear Algebra: A Modern Introduction by Poole.

- Vancouver Community College: Costa Karavas.
  Text: Linear Algebra and its Applications, 5th edition, by Lay et al.

7.3 Summary Data

The following tables contain counts: how many times institutions rated a given topic as 'core,' 'additional,' or 'omit.'

Systems of Equations		Science Core	Add'l	Omit		Engineering Core	Add'l	Omit
1.1	Systems of Equations, Geometry	26	0	0	C	9	0	0
1.2	Systems of Equations, Algebraic Procedures	27	0	0	C	9	0	0
1.2.1	Elementary Operations	27	0	0	C	9	0	0
1.2.2	Gaussian Elimination	27	0	0	C	9	0	0
1.2.3	Uniqueness of Reduced Row-Echelon Form	18	8	1	A	4	3	2
1.2.4	Rank and Homogeneous Systems	24	3	0	C	9	0	0
Matrices								
2.1	Matrix Arithmetic	27	0	0	C	9	0	0
2.1.1	Addition of Matrices	27	0	0	C	9	0	0
2.1.2	Scalar Multiplication of Matrices	27	0	0	C	9	0	0
2.1.3	Multiplication of Matrices	27	0	0	C	9	0	0
2.1.5	Properties of Matrix Multiplication	27	0	0	C	9	0	0
2.1.6	The Transpose	25	2	0	C	6	3	0
2.1.7	The Identity and Inverses	27	0	0	C	9	0	0
2.1.8	Finding the Inverse of a Matrix	27	0	0	C	9	0	0
2.1.9	Elementary Matrices	14	13	0	A	3	6	0
2.1.10	Properties of Invertible Matrices	25	2	0	C	7	2	0

		Science				Engineering		
Determinants								
3.1	Basic Techniques and Properties	27	0	0	C	9	0	0
3.1.1	Cofactors and 2×2 Determinants	27	0	0	C	9	0	0
	Laplace's Formula (cofactor expansion in higher dimensions)	26	1	0	C	9	0	0
3.1.2	The Determinant of a Triangular Matrix	24	3	0	C	6	3	0
3.1.3	Properties of Determinants	26	1	0	C	6	3	0
	Computing a determinant by permutation expansion	1	11	15	O	1	1	7
	The only alternating, multilinear form taking the value 1 on the identity	1	7	18	O	0	1	8
3.1.4	Finding Determinants using Row Operations	20	6	1	C	4	4	1
3.2	Applications of the Determinant	20	7	0	A	6	2	1
3.2.1	A Formula for the Inverse	12	13	2	A	1	7	1
3.2.2	Cramer's Rule	11	13	3	A	3	5	1

$\mathbb{R}^n$		Science Core	Science Add'l	Science Omit		Engineering Core	Engineering Add'l	Engineering Omit
4.1	Vectors in $\mathbb{R}^n$	27	0	0	C	9	0	0
4.2	Algebra in $\mathbb{R}^n$	27	0	0	C	9	0	0
4.2.1	Addition of Vectors in $\mathbb{R}^n$	27	0	0	C	9	0	0
4.2.2	Scalar Multiplication of Vectors in $\mathbb{R}^n$	27	0	0	C	9	0	0
4.3	Geometric Meaning of Vector Addition	25	2	0	C	9	0	0
4.4	Length of a Vector	24	3	0	C	9	0	0
4.5	Geometric Meaning of Scalar Multiplication	24	3	0	C	9	0	0
4.6	Parametric Lines	18	8	1	A	8	1	0
4.7.1	The Dot Product	26	1	0	C	9	0	0
4.7.2	The Geometric Significance of the Dot Product	20	7	0	A	9	0	0
4.7.3	Projections	19	7	1	A	9	0	0
4.8	Planes in $\mathbb{R}^n$	19	8	0	A	7	1	1
4.9	The Cross Product	14	9	4	A	6	0	3
4.9.1	The Box Product	5	10	12	O	1	5	3
4.10	Spanning, Linear Independence and Basis in $\mathbb{R}^n$	25	1	1	C	7	1	1
4.10.1	Spanning Set of Vectors	25	1	1	C	7	1	1
4.10.2	Linearly Independent Set of Vectors	25	1	1	C	7	1	1
4.10.4	Subspaces and Basis	25	1	1	C	7	0	2
4.10.5	Row Space, Column Space, and Null Space of a Matrix	22	4	1	C	5	2	2
4.11	Orthogonality and the Gram Schmidt Process	17	6	4	A	5	3	1
4.11.1	Orthogonal and Orthonormal Sets	17	6	3	A	5	4	0
4.11.2	Orthogonal Matrices	8	15	4	A	3	5	1
4.11.3	Gram-Schmidt Process	13	10	3	A	5	3	1
4.11.4	Orthogonal Projections	14	9	3	A	4	5	0
4.11.5	Least Squares Approximation	3	16	7	A	1	4	4

			Science				Engineering	
Linear Transformations		Core	Add'l	Omit		Core	Add'l	Omit
5.1	Linear Transformations	25	1	1	C	8	0	1
5.2	The Matrix of a Linear Transformation	25	1	1	C	8	0	1
5.3	Properties of Linear Transformations	25	1	1	C	8	0	1
5.4	Special Linear Transformations in the Euclidean Plane	15	9	3	A	7	1	1
5.5	Linear Transformations which are One-to-One or Onto	18	6	3	A	4	1	4
5.6	Isomorphisms	9	11	7	A	1	2	6
5.7	The Kernel And Image Of A Linear Map	18	7	2	A	2	4	3
5.9	The General Solution of a Linear System	19	5	3	C	5	3	1
	Changes of Basis in $\mathbb{R}^n$	13	9	4	A	2	2	5
Complex Numbers		Core	Add'l	Omit		Core	Add'l	Omit
6.1	Complex Numbers	11	4	11	O	9	0	0
6.2	Polar Form	8	6	12	O	8	1	0
6.3	Roots of Complex Numbers	4	8	14	O	6	1	2
6.4	The Quadratic Formula	6	10	10	A	6	2	1

Spectral Theory

		Science				Engineering		
7.1	Eigenvalues and Eigenvectors of a Matrix	26	1	0	C	9	0	0
7.1.1	Definition of Eigenvectors and Eigenvalues	26	1	0	C	9	0	0
7.1.2	Finding Eigenvectors and Eigenvalues	26	1	0	C	9	0	0
7.1.3	Eigenvalues and Eigenvectors for Diagonal Matrices	20	7	0	A	8	1	0
	Eigenvalues and Eigenvectors for Similar Matrices	16	10	0	A	4	4	0
7.2	Diagonalization	23	3	1	C	8	1	1
7.2.1	Diagonalizing a Matrix	23	3	1	C	8	1	1
7.2.2	Complex Eigenvalues	5	11	10	A	7	2	10
7.3	Applications of Spectral Theory	6	11	9	A	3	2	9
7.4	Orthogonality	5	9	12	O	2	2	12
7.4.1	Orthogonal Diagonalization	5	8	13	O	2	2	13
7.4.2	Positive Definite Matrices	2	8	16	O	1	2	16
7.4.3	QR Factorization	0	12	14	O	0	4	14
7.4.4	Quadratic Forms	0	10	16	O	0	3	16

		Science				Engineering		
Vector Spaces								
9.1	Algebraic Considerations	18	6	3	A	2	3	4
9.2	Subspaces	19	4	4	C	3	1	5
9.3	Linear Independence and Bases	20	3	4	C	3	2	4
	Isomorphisms	6	9	11	O	0	2	7
	Homomorphisms	3	10	13	O	0	2	7
	Range and Null Spaces	16	6	5	A	1	2	6
	Change of Basis	14	7	5	A	2	2	5
	Changing Representations of Vectors	11	10	5	A	1	3	5
	Changing Map Representations	4	10	12	O	0	2	7
Inner Product Spaces, Proofs								
	Axioms	7	7	12	O	1	2	6
	Length, Angle and Orthogonality	7	7	12	O	1	2	6
	Least Squares	1	10	15	O	0	3	6
	L^2-Spaces	0	7	19	O	1	2	6
	Students must write simple proofs throughout the course	21	5	1	C	2	3	4

Appendices

A Counts of Topics for Each Institution

Institution	Core	Additional	Omit
SFU	69	12	12
UBCV	47	38	8
UNBC	58	1	9
UVic	67	12	14
UAlb	51	6	35
UCal	55	4	34
ULeth	41	31	21
CapU	47	32	14
KPU	60	10	23
TRU	61	26	5
TRUO	70	8	15
UBCO	43	14	36
UFV	66	9	18
VIU	56	33	4
Alex	71	15	6
Cam	79	2	12
CNC	49	31	13
CotR	79	14	0
ColC	67	4	22
CoqC	70	10	13
Doug	71	10	12
Lang	50	29	14
NIC	56	17	20
NLC	49	40	4
OC	49	41	3
Selk	67	24	2
VCC	71	19	3
Averages:	70.4	21.4	16.2

B Topics Omitted from Science Linear Algebra.

Determinants

Computing a determinant by permutation expansion

The only alternating, multilinear form taking the value 1 on the identity

$\mathbb{R}^n$

4.9.1 The Box Product

Complex Numbers

6.1 Complex Numbers

6.2 Polar Form

6.3 Roots of Complex Numbers

Spectral Theory

7.4 Orthogonality

7.4.1 Orthogonal Diagonalization

7.4.2 Positive Definite Matrices

7.4.3 QR Factorization

7.4.4 Quadratic Forms

Vector Spaces

Isomorphisms

Homomorphisms

Changing Map Representations

Inner Product Spaces, Proofs

Axioms

Length, Angle and Orthogonality

Least Squares

L^2-Spaces

C Texts in Use

Author	Used by
Howard Anton	ColC, KPU, Lang, NLC
Jeffrey Holt	UBCO
Ken Kuttler	CotR, UCal
Ron Larson	Cam, TRUO, VIU
David C. Lay	Alex, CoqC, Doug, UBCV, OC, VCC, SFU
Steve Leon	UFV
Keith Nicholson	Selk, UNBC
Norman, Wolczuk	NIC, UVic
David Poole	CapU, CNC, TRU, UAlb

D The Form

The form which was filled out by the universities and colleges has been appended to this document.

Data for a Core Linear Algebra Agreement

Jim Bailey, Nora Franzova, Claude Laflamme,
Michael Nyenhuis, Wesley Snider.

This list of topics is the table of contents from A First Course in Linear Algebra by Ken Kuttler.

Institutions: Your Institution

Contact Person:

Course Type: Science Hours per week: 3

Course Identifier: (such as MATH 221)

Text:

Author:

Instructions:

- Please try to fill out the form so that it approximates your department's consensus on what should be in this course.
- You will need a recent version of Adobe Acrobat Reader. Fill in part of the form, then save and reload it. Some earlier versions of Adobe Acrobat Reader will not let you save a form if it has been filled in.
- Interpret the Course Type as follows:

  Science is the course which the majority of students take;

  Engineering has fewer topics and applications are from applied science;

  Mathematics is intended for mathematics majors; proofs are emphasised.

  If your institution offers more than one type of Linear Algebra, then please fill in a separate form for each one.
- Please use the following interpretations:
  - Core: topics which must be taught and take approximately 75% of the course.
  - Additional: a list of topics which need not be taught, but a subset of them should be taught for breadth.
  - Omit: this topic is not important; it should be left out of the analysis.
- if a drop-down-menu does not contain the option which you need, then you can type it in.
- topics without a section number are not in Kuttler.

Where should the following topics be placed? Core Additional Omit

Systems of Equations

		Core	Additional	Omit
1.1	Systems of Equations, Geometry	◯	◯	◯
1.2	Systems of Equations, Algebraic Procedures	◯	◯	◯
1.2.1	Elementary Operations	◯	◯	◯
1.2.2	Gaussian Elimination	◯	◯	◯
1.2.3	Uniqueness of Reduced Row-Echelon Form	◯	◯	◯
1.2.4	Rank and Homogeneous Systems	◯	◯	◯

Matrices

		Core	Additional	Omit
2.1	Matrix Arithmetic	◯	◯	◯
2.1.1	Addition of Matrices	◯	◯	◯
2.1.2	Scalar Multiplication of Matrices	◯	◯	◯
2.1.3	Multiplication of Matrices	◯	◯	◯
2.1.5	Properties of Matrix Multiplication	◯	◯	◯
2.1.6	The Transpose	◯	◯	◯
2.1.7	The Identity and Inverses	◯	◯	◯
2.1.8	Finding the Inverse of a Matrix	◯	◯	◯
2.1.9	Elementary Matrices	◯	◯	◯
2.1.10	Properties of Invertible Matrices	◯	◯	◯

Determinants

		Core	Additional	Omit
3.1	Basic Techniques and Properties	◯	◯	◯
3.1.1	Cofactors and 2×2 Determinants	◯	◯	◯
Laplace's Formula (cofactor expansion in higher dimensions)		◯	◯	◯
3.1.2	The Determinant of a Triangular Matrix	◯	◯	◯
3.1.3	Properties of Determinants	◯	◯	◯
Computing a determinant by permutation expansion		◯	◯	◯
The only alternating, multilinear form taking the value 1 on the identity		◯	◯	◯
3.1.4	Finding Determinants using Row Operations	◯	◯	◯
3.2	Applications of the Determinant	◯	◯	◯
3.2.1	A Formula for the Inverse	◯	◯	◯
3.2.2	Cramer's Rule	◯	◯	◯

Where should the following topics be placed?
$\mathbb{R}^n$

		Core	Additional	Omit
4.1	Vectors in $\mathbb{R}^n$	○	○	○
4.2	Algebra in $\mathbb{R}^n$	○	○	○
4.2.1	Addition of Vectors in $\mathbb{R}^n$	○	○	○
4.2.2	Scalar Multiplication of Vectors in $\mathbb{R}^n$	○	○	○
4.3	Geometric Meaning of Vector Addition	○	○	○
4.4	Length of a Vector	○	○	○
4.5	Geometric Meaning of Scalar Multiplication	○	○	○
4.6	Parametric Lines	○	○	○
4.7.1	The Dot Product	○	○	○
4.7.2	The Geometric Significance of the Dot Product	○	○	○
4.7.3	Projections	○	○	○
4.8	Planes in $\mathbb{R}^n$	○	○	○
4.9	The Cross Product	○	○	○
4.9.1	The Box Product	○	○	○
4.10	Spanning, Linear Independence and Basis in $\mathbb{R}^n$	○	○	○
4.10.1	Spanning Set of Vectors	○	○	○
4.10.2	Linearly Independent Set of Vectors	○	○	○
4.10.4	Subspaces and Basis	○	○	○
4.10.5	Row Space, Column Space, and Null Space of a Matrix	○	○	○
4.11	Orthogonality and the Gram Schmidt Process	○	○	○
4.11.1	Orthogonal and Orthonormal Sets	○	○	○
4.11.2	Orthogonal Matrices	○	○	○
4.11.3	Gram-Schmidt Process	○	○	○
4.11.4	Orthogonal Projections	○	○	○
4.11.5	Least Squares Approximation	○	○	○

Where should the following topics be placed? Core Additional Omit

Linear Transformations

		Core	Additional	Omit
5.1	Linear Transformations	◯	◯	◯
5.2	The Matrix of a Linear Transformation	◯	◯	◯
5.3	Properties of Linear Transformations	◯	◯	◯
5.4	Special Linear Transformations in the Euclidean Plane	◯	◯	◯
5.5	Linear Transformations which are One-to-One or Onto	◯	◯	◯
5.6	Isomorphisms	◯	◯	◯
5.7	The Kernel And Image Of A Linear Map	◯	◯	◯
5.9	The General Solution of a Linear System	◯	◯	◯
Changes of Basis in $\mathbb{R}^n$		◯	◯	◯

Complex Numbers

		Core	Additional	Omit
6.1	Complex Numbers	◯	◯	◯
6.2	Polar Form	◯	◯	◯
6.3	Roots of Complex Numbers	◯	◯	◯
6.4	The Quadratic Formula	◯	◯	◯

Spectral Theory

		Core	Additional	Omit
7.1	Eigenvalues and Eigenvectors of a Matrix	◯	◯	◯
7.1.1	Definition of Eigenvectors and Eigenvalues	◯	◯	◯
7.1.2	Finding Eigenvectors and Eigenvalues	◯	◯	◯
7.1.3	Eigenvalues and Eigenvectors for Diagonal Matrices	◯	◯	◯
	Eigenvalues and Eigenvectors for Similar Matrices	◯	◯	◯
7.2	Diagonalization	◯	◯	◯
7.2.1	Diagonalizing a Matrix	◯	◯	◯
7.2.2	Complex Eigenvalues	◯	◯	◯
7.3	Applications of Spectral Theory	◯	◯	◯
7.4	Orthogonality	◯	◯	◯
7.4.1	Orthogonal Diagonalization	◯	◯	◯
7.4.2	Positive Definite Matrices	◯	◯	◯
7.4.3	QR Factorization	◯	◯	◯
7.4.4	Quadratic Forms	◯	◯	◯

Where should the following topics be placed?	Core	Additional	Omit
Vector Spaces			
9.1 Algebraic Considerations	◯	◯	◯
9.2 Subspaces	◯	◯	◯
9.3 Linear Independence and Bases	◯	◯	◯
Isomorphisms	◯	◯	◯
Homomorphisms	◯	◯	◯
Range and Null Spaces	◯	◯	◯
Change of Basis	◯	◯	◯
Changing Representations of Vectors	◯	◯	◯
Changing Map Representations	◯	◯	◯

Inner Product Spaces

Axioms	◯	◯	◯
Length, Angle and Orthogonality	◯	◯	◯
Least Squares	◯	◯	◯
L^2-Spaces	◯	◯	◯

Introduction to Proofs

Students must write simple proofs throughout the course	◯	◯	◯

What are the prerequisites for this course?	Other
Must your students use software?	No
What software do you use?	Other software
Are labs compusory?	Yes ☐

Comments: please include any topics which you would like to have added. If there is not enough room, then please continue when you email the form.

Save and email the completed form to Jim Bailey, bailey@cotr.bc.ca.
Version: June 9, 2016.