Science Calculus Report 2019 08 08

Prior to the annual British Columbia Committee on the Undergraduate Program in Mathematics and Statistics (BCCUPMS) meeting in May 2019, the Core Calculus Committee formulated two surveys, one of 205 questions for Science Calculus, and one of 164 questions for Business/Social Sciences Calculus. The surveys were completed in the month after the annual meeting. Twenty-two institutions responded to the science survey, and 18 responded to the Business Calculus survey.

Science Calculus was taken to mean the course intended for future mathematicians, physicists, chemists and engineers. Some institutions have a calculus for life sciences, and some of these filled in the survey assuming "science calculus" meant either of these. Conversely, at some institutions "Business Calculus" is Calculus for Life Science, Business, and Social Sciences.

Each survey consisted of a list of learning outcomes, and respondents were to identify whether the learning outcome was taught in Calculus I, Calculus II, or neither, and whether the learning outcome was core, optional, or whether it could be omitted. We interpreted "core" to mean that the learning outcome should be in any calculus course intended for mathematicians etc, though some respondents interpreted it as meaning that the learning outcome was part of the Core Calculus Curriculum[1] (CCC) developed by the BCCUPMS in 2002.

In order to analyze the data we categorized the learning outcomes, and each outcome could belong to more than one category. The categories were: limits, derivatives, differential applications (tangent line, differentials, Newton's Method, parametric curves, related rates), sketching, optimization, integrals, integration applications, DEs, series (and sequences), and curves (polar, parametric, conic).

Institutions were grouped as "all," and "B.Sc." (of which there were 8), where "B.Sc." meant any institution that was offering a B.Sc. in Mathematics by the summer of 2019. The argument for singling out responses from this group is that students interested in math are likely to transfer to institutions with a math B.Sc, so the opinions of these institutions should be taken into account separately.

We end the summary of each category with a possible learning outcome. These are based on what was identified as core by all institutions, tempered by what was identified as core by the B.Sc. institutions, and by what is core in the CCC. We have not included recommendations from the Engineering Common Core Final Report[2], prepared for the British Columbia Committee on Articulation and Transfer by the engineering articulation group, but they endorse the CCC, with the added recommendation that parametric curves; series, including integral, comparison, limit comparison, alternating series and root tests; polar coordinates; and complex numbers be taught as well. The possible learning outcomes, combined, do satisfy the CCC, and, if fact, add detail to the learning outcomes listed there.

Results

Anomalies

There were unexpected responses:

- Some questions were left completely blank. This was interpreted to mean that the respondent was not sure if the learning outcome was taught in Calculus I or II, and had no opinion on whether it

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[1] *Final Report: First Year Core Calculus*, downloaded August 8, 2019 from http://www.bccat.ca/pubs/calculus.pdf

[2] *First-Year Common Engineering Curriculum for the BC Post-Secondary Sector – Implementation Phase: Final Report*, downloaded August 8, 2019 from https://www.bccat.ca/pubs/Engineering%20Common%20Core%20Final%20Report%20with%20amendments%20Nov%2013%202018-%20v113.pdf

was core. It was felt that such non-responses could be lumped in with "not core" responses (that is the combined optional and omit responses), and not taught in either Calculus I or II.

- Some respondents checked off both Calculus I and II. This was interpreted to mean that the learning outcome was found in both courses.
- Some respondents identified a learning outcome as being in Calculus I or II, but did not indicate whether the learning outcome was core. This was interpreted to mean that the respondent was not sure of the departmental consensus.
- Effort was made to clarify impossible responses, such as a learning outcome being taught in no course, yet being core, which were then corrected as need be.

These unexpected responses, different interpretations of what the learning outcomes mean, and the fact that some respondents were answering the survey for two different courses mean that the numbers should be taken as indications of what is taught and what is core, but should not be relied on completely.

Learning Outcomes by Course

Institutions in B.C. tend to teach the same learning outcomes in Calculus I. Most institutions did not teach a significant amount of integration is Calculus I, though 6 institutions on Vancouver Island and close to the Alberta border did. The University of Victoria and the Alberta universities teach integration in the first term. Time is made for integration by shifting some material, such as L'Hospital's Rule and parametric curves, that other institutions teach in Calculus I, to Calculus II. Typically, limits, differentiation, differential applications, sketching, and optimization are taught in Calculus I.

Most learning outcomes were identified as "core" by a large fraction of institutions, and very few were identified as "omit." It is reasonable, therefore, to combine the "optional," "omit," and "did not answer" categories as a single "non-core" category. For analysis purposes, we assumed that if 17 (77%) or more institutions agreed on whether a learning outcome was core, or, conversely, if 6 (27%) or fewer felt it was core, then the outcome was identified broadly as core, or, conversely, not core. Learning outcomes that were identified as core by between 7 and 16 institutions were identified limitedly as core.

Limits

All learning outcomes other than 5: epsilon-delta proofs, and 11: interpreting limits in an applied context were identified broadly as "core."

All learning outcomes are taught in Calculus I, with the exception of L'Hospital's Rule, which is taught in Calculus II at some institutions.

A Possible Learning Outcome

We expect students to be able to take limits, including improper limits and one-sided limits. We expect that they can take limits of algebraic functions, and expressions involving transcendental functions. We expect that they are able to take limits of composed functions and to use the Squeeze Theorem. We expect that students can determine continuity of functions at points and on closed intervals, and that they are able to apply the Intermediate Value Theorem. Those who do not teach integration in Calculus I expect students to be able to apply L'Hospital's Rule.

It should be pointed out that we omitted to ask about piecewise functions. We tacitly assumed that one-sided limits necessarily required that piecewise functions be covered.

Derivatives

Almost all learning outcomes were identified broadly as core. Those identified broadly as not core were those involving hyperbolic functions (outcomes 103–106). Learning outcomes identified as neither broadly core nor not core are: 29: finding a derivative from the graph, 36: simplifying derivatives, 44: finding the derivative of an inverse function, 54: Rolle's Theorem, and 200: finding the slope of a parametric curve. These learning outcomes were identified limitedly as core by B.Sc. institutions, with the exception finding the derivative of an inverse function. It should be noted that simplifying a derivative is not necessarily taught in calculus, but many institutions do require it of students.

A Possible Learning Outcome

We expect that students be able to find derivatives, using the product, quotient and chain rules, and involving algebraic and transcendental functions. Students are expected to differentiate functions involving logarithms and exponentials with any base, and inverse trig functions. We require that they be able to differentiate implicit functions, find the derivatives of inverse functions, and do logarithmic differentiation. We require that students be able to use the Mean Value Theorem.

Sketching and Optimization

All learning outcomes were identified broadly as core.

Possible Learning Outcomes

We expect that students be able to find horizontal and vertical asymptotes, intervals of increase and decrease, concavity, inflection points, and be able to use this information to sketch the graph of the function.

We expect that students be able to formulate an optimization problem described in words, find and classify local extrema using the First and Second Derivative Tests, and that they be able to find and classify global extrema on closed and non-closed intervals.

Applications of Differentiation

Learning outcomes identified broadly as core are 47: related rates, 67: tangent line approximation, and 70: differentials. Those identified broadly as not core are 49: demonstrating when Newton's Method fails, 69: estimating errors using differentials, 199: the tautochrone and brachistochrone problems, and 214: Kepler's Laws of Planetary Motion. Outcomes identified limitedly as core were 48: Newton's Method, and 68: comparing differentials and actual differences. The B.Sc. institutions agreed with these assessments.

A Possible Learning Outcome

We expect that students are able to solve real-life related rates problems, apply Newton's Method to find roots, use the tangent line approximation, and find differentials of functions.

Integration in Calculus I

Some institutions do a very short introduction to anti-derivatives, while six institutions (Camosun College, College of the Rockies, North Island College, Northern Lights College, Selkirk College, and University of Victoria) do a more thorough introduction, including the Riemann sum definition of integral, sigma notation, basic integrals and substitution. These institutions are of two types, those on Vancouver Island, who presumably expect students to transfer to University of Victoria, and those close to the Alberta border, who presumably expect students to transfer to an Alberta institution. Those close to the Alberta border tend to cover the Riemann sum definition of integration, sigma notation, areas between curves, and substitution, though the amount of coverage varies. They usually cover L'Hospital's Rule in

Calculus I, and parametric curves in Calculus II. Those on the island cover the Riemann sum definition of integral, sigma notation, and areas between curves. At least one does numerical integration. They tend to cover L'Hospital's Rule and parametric curves in Calculus II. Note that the two groups consist of three institutions each, so factoring in how survey questions were interpreted, our observations should not be taken as definitive.

As should be clear from the foregoing, institutions that do not cover integration in Calculus I tend to do L'Hospital's Rule and parametric curves in Calculus I.

Integration

The responses for institutions teaching integration in Calculus I, Calculus II and both were combined to determine "coreness." Learning outcomes identified broadly as not core were 97: analyzing the error in the Trapezoid and Simpson's Rules, 100: integration of functions involving arccsc, arcsec and arccot, hyperbolic functions (103--106), 154: using tables to find integrals, and 155: reduction formulae. These learning outcomes were also identified broadly as not core by the B.Sc. institutions, as was 148: integrating products of sines and cosines with different angles. Learning outcomes identified limitedly as core were 78: finding the area of a plane region using limits, 83: Mean Value Theorem for integrals, 87: approximating an integral from its graph, 91: evaluating the integral of even and odd functions, 94–96: numerical integration, 98: evaluating integrals using the log rule, 101: evaluating integrals by completing the square, 102: review of techniques of integration, 128: viewing integrals as a process of accumulation, 147: integrals of products of secants and tangents, 148: products of sine and cosine with different angles, and 156: rational functions of sines and cosines. Note that partial fractions with quadratic factors was viewed broadly as core, as was products of sines and cosines, as opposed to rational functions of sines and cosines. B.Sc. Institutions largely agreed, though they identified broadly as core the Midpoint and Trapezoid Rules, and viewing integrals as an accumulation process.

It should be noted that we did not ask whether estimation of convergence and divergence of integrals was core. Convergence and divergence of integrals is "core" in the CCC.

A Possible Learning Outcome

We expect students to be able to define and evaluate an integral using Riemann sums and sigma notation, use the Fundamental Theorem of Calculus, evaluate integrals using substitution, integration by parts, partial fractions, including with those whose denominators have quadratic factors. We expect them to integrate products of trig functions, and to do inverse trig substitution. We expect them to evaluate improper integrals, establish convergence or divergence of improper integrals, and to be able to use the Midpoint and Trapezoid Rules.

Applications of Integration

The learning outcomes identified broadly as core were 77, 126, 127: finding the area between curves, 129–131: finding volumes of rotation by disks and washers, and 131: Finding the volume of a solid with known cross-sections. Outcomes identified broadly as not core were 138–142 masses and centers of mass, 143: fluid pressure, 201, 201: arclength of parametric and polar curves, 202, 211, 212 surface areas of surfaces of revolution in parametric and polar form, and 208 area of a polar graph. The remaining outcomes (93: finding position and velocity of a particle by integration, 84: average values of functions, 132: shells, 134: arclength, 135: surface area of revolution, 136–137 work, 150: using integrals to model real-life applications, 185: integrating and differentiating power series) were identified limitedly as core, though 93, 132, 134, and 150 were borderline (16 core). B.Sc. institutions agreed with these assessments, though 93, 132, 136, 137, and 150 were identified broadly as core, and 135 was identified broadly as not core.

A Possible Learning Outcome

We expect students to be able to find areas between curves, find volumes of solids with known cross-section, by slices and cylindrical shells, and determine work done.

Differential Equations

Only the basic differential equation learning outcomes were broadly seen as core. Specifically, 74, 109: solving initial value problems, 108: checking that a solution solves a DE, and 113, 116: solving separable DEs. The only learning outcomes identified limitedly as core were 114, 115: using DE's (resp. exponentials) to model real-life situations. The remainder were identified broadly as not core. B.Sc. institutions agreed with these assessments, except that 74 was identified limitedly as core.

A Possible Learning Outcome

We expect students to be able check that a function solves a differential equation, and to solve simple differential equations, including separable ones.

Sequences and Series

Outcomes identified broadly as core were 162: listing the terms of a sequence, 163: writing a formula for the n^{th} term, 164: determining convergence of a sequence, 166, 167: convergence of infinite and geometric series, 168: n^{th} term test for divergence of a series, 176: ratio test, 178: applying convergence tests for series, understanding and finding power series (179, 180, 182, 186, 188), and 183: finding the radius of convergence. Only 189: binomial series was identified broadly as not core. Ones identified limitedly as core were 165: using properties of monotonic sequences to establish convergence or divergence, 169–173, 177: integral, p-series, direct and limit comparison, alternating series tests, root test, 174: remainder term for the alternating series test, 175: absolute and conditional convergence, 181: the remainder term for Taylor polynomials, 184: determining endpoint convergence, 185: differentiating and integrating power series, and 187: generating new series from old. The B.Sc. institutions largely agreed, though they identified broadly as core differentiation and integration of power series.

Note that the only real convergence test identified as core was the ratio test, yet determining convergence of power series was identified as core.

A Possible Learning Outcome

We expect students to be able to find the Taylor series of a function using the definition, find the radius of convergence, and understand what convergence means, and to use the Taylor series to approximate function values.

Curves

All learning outcomes in the curves section were identified broadly as not core, with the exception of 196: sketching parametric curves and 200: finding the slope of the tangent to a parametric curve, both of which were identified limitedly as core. B.Sc. institutions agreed with these assessments.

Data

Triples under All and B.Sc. indicate the number of "core," "optional" and "omit" responses. Blank responses indicate no response.

Limits

Course	No.		All	B.Sc.
Calc I	2	Estimating a limit using a numerical or graphical approach	20, 1, 0	8, 0,0
	3	Determining one-sided limits	21, 0,0	7, 0,0
	4	Different ways that a limit can fail to exist	21, 1,0	8, 0,0
	5	Studying and using of the epsilon-delta definition of limit	2, 6,0	0, 1,0
	6	Evaluating limits using properties of limits	22, 0,0	8, 0,0
	7	Developing and using a strategy for finding limits	19, 1,0	7, 1,0
	8	Evaluating a limit using the dividing out technique (dominating power analysis)	22, 0,0	8, 0,0
	9	Evaluating a limit using the rationalizing technique	22, 0,0	8, 0,0
	10	Evaluating a limit using the Squeeze Theorem	17, 3,0	6, 1,0
	11	Interpreting in everyday language the meaning of a limiting value in an applied context	13, 7,0	7, 1,0
	12	Determining continuity at a point and continuity on an open interval	21, 0,0	7, 0,0
	13	Determining continuity on a closed interval	19, 2,0	7, 0,0
	14	Using the limit properties of continuity to establish limits of function compositions and combinations	18, 3,0	6, 1,0
	15	Demonstrating an understanding of the Intermediate Value Theorem	19, 3,0	7, 1,0
	16	Determining end behavior of functions - algebraic and transcendental	19, 2,0	7, 0,0
	17	Finding and sketching the vertical asymptotes of the graph of a function	20, 2,0	8, 0,0
	61	Determining (finite) limits at infinity	22, 0,0	8, 0,0
	62	Determining the horizontal asymptotes, if any, of the graph of a function	19, 2,0	6, 1,0
	63	Determining infinite limits at infinity	20, 0,0	7, 0,0
	157	Recognizing limits that produce indeterminate forms	11, 1,0	5, 1,0
	158	Applying L'Hospital's Rule to evaluate a limit	12, 2,0	5, 2,0
Calc II	157	Recognizing limits that produce indeterminate forms	9, 1,0	2, 0,0
	158	Applying L'Hospital's Rule to evaluate a limit	7, 1,0	1, 0,0

Derivatives

Course	No.	Question	All	B.Sc.
Calc I	19	Finding the slope of the tangent line to a given function at a specified input value	22, 0,0	8, 0,0
	20	Using the limit definition to find the derivative of a function	21, 0,0	8, 0,0
	21	Understanding the relationship between differentiability and continuity	20, 2,0	7, 1,0
	22	Finding the derivative of a function using the Constant Rule	22, 0,0	8, 0,0
	23	Finding the derivative of a function using the Power Rule	22, 0,0	8, 0,0
	24	Finding the derivative of a function using the Constant Multiple Rule	22, 0,0	8, 0,0
	25	Finding the derivative of a function using the Sum and Difference Rules	22, 0,0	8, 0,0
	26	Finding the derivatives of the sine function and of the cosine function	22, 0,0	8, 0,0
	27	Finding the derivatives of exponential functions	22, 0,0	8, 0,0
	28	Using derivatives to find rates of change	22, 0,0	8, 0,0
	29	Approximating derivatives from the graph	13, 6,0	5, 2,0
	30	Finding the derivative of a function using the Product Rule	22, 0,0	8, 0,0
	31	Finding the derivative of a function using the Quotient Rule	22, 0,0	8, 0,0
	32	Finding the derivative of the tangent, cotangent, secant, and cosecant functions	21, 1,0	7, 1,0
	33	Finding a higher-order derivative of a function	21, 1,0	8, 0,0
	34	Finding the derivative of a composite function using the Chain Rule	22, 0,0	8, 0,0
	35	Finding the derivative of a function using the General Power Rule	18, 3,0	5, 2,0
	36	Simplifying the derivative of a function using algebra	13, 4,1	4, 2,0
	37	Finding the derivative of a composition involving a transcendental function	22, 0,0	8, 0,0
	38	Finding the derivative of a function involving the natural logarithmic function	22, 0,0	8, 0,0
	39	Defining and differentiating exponential functions that have bases other than e	20, 2,0	8, 0,0
	40	Interpreting in everyday language the meaning of a numerical derivative value in an applied context	17, 4,0	7, 1,0
	41	Distinguishing between functions written in implicit form and explicit form	21, 0,0	7, 0,0
	42	Using implicit differentiation to find the derivative of a function	22, 0,0	8, 0,0
	43	Finding derivatives of functions using logarithmic differentiation	17, 5,0	7, 1,0
	44	Finding the derivative of an inverse function	15, 5,0	7, 1,0
	45	Differentiating an inverse trigonometric function	20, 1,0	8, 1,0
	46	Applying implicit differentiation to a variable relationship to establish a relationship between their rates of change	18, 2,0	7, 1,0
	54	Demonstrating an understanding of the Rolle’s Theorem	14, 6,0	5, 2,0
	55	Demonstrating an understanding of the the Mean Value Theorem	18, 4,0	7, 1,0

	103	Developing properties of hyperbolic functions	0, 3,0	0, 1,0
	104	Differentiation and integration of hyperbolic functions	0, 1,0	0, 0, 0
	105	Developing properties of inverse hyperbolic functions	0, 1,0	0, 0, 0
	106	Differentiation and integration of functions involving inverse hyperbolic functions	0, 1,0	0, 0, 0
	200	Finding the slope of a tangent line to a curve given by a set of parametric equations	7, 2, 0	3, 1, 0
Calc II	45	Differentiating an inverse trigonometric function	1, 0,0	0, 0, 0
	103	Developing properties of hyperbolic functions	3, 3,0	1, 2,0
	104	Differentiation and integration of hyperbolic functions	1, 5,0	0, 2,0
	105	Developing properties of inverse hyperbolic functions	1, 3,0	0, 1,0
	106	Differentiation and integration of functions involving inverse hyperbolic functions	1, 2,0	0, 1,0
	200	Finding the slope of a tangent line to a curve given by a set of parametric equations	3, 3,0	0, 1,0

Sketching

Course	No.		All	B.Sc.
Calc I	17	Finding and sketching the vertical asymptotes of the graph of a function	20, 2,0	8, 0,0
	56	Determining intervals on which a function is increasing or decreasing	22, 0,0	8, 0,0
	58	Determining intervals on which a function is concave upward or concave downward	22, 0,0	8, 0,0
	59	Finding any points of inflection of the graph of a function	22, 0,0	8, 0,0
	64	Analyzing and sketching the graph of a function using information form the first and second derivatives	18, 4,0	6, 2,0

Optimization

Course	No.	Question	All	B.Sc.
Calc I	51	Understanding the definition of local and global extrema of a function on an interval	22, 0,0	8, 0,0
	52	Understanding the definition of local extrema of a function on an open interval	22, 0,0	8, 0,0
	53	Finding extrema on a closed interval	22, 0,0	8, 0,0
	57	Applying the First Derivative Test to find relative extrema of a function	22, 0,0	8, 0,0
	60	Applying the Second Derivative Test to find relative extrema of a function	18, 4,0	7, 1,0
	65	Generating an objective function and domain in an applied optimization problem	20, 1,0	8, 0,0

	No.	Question	All	B.Sc.
	66	Finding the absolute max or min of a given function in an applied setting	20, 2,0	8, 0,0

Applications of Differentiation

Course	No.	Question	All	B.Sc.
Calc I	47	Using related rates to solve real-life problems	19, 2,0	7, 1,0
	48	Approximating a zero of a function using Newton's method	12, 10,0	4, 4,0
	49	Demonstrating failures of Newton's method graphically and algebraically	6, 13,0	2, 5,0
	67	Understanding the concept of a tangent line approximation	19, 3,0	8, 0,0
	68	Comparing the value of the differential, dy, with the actual change in y, Delta y	13, 6,0	3, 3,0
	69	Estimating a propagated error using a differential	5, 11,0	2, 4,0
	70	Finding the differential of a function using differentiation formulas	17, 2,0	5, 1,0
	199	Understanding two classic calculus problems, the tautochrone andbrachistochrone problems	0, 1,0	0, 1,0
Calc II	199	Understanding two classic calculus problems, the tautochrone andbrachistochrone problems	0, 2,0	
	214	Understanding and using Kepler's Laws of planetary motion	0, 1,0	

Integrals

Course	No.		All	B.Sc.
Calc I	72	Interpreting an indefinite integral as an antiderivative	8, 0,0	1, 0, 0
	73	Using basic rules of antidifferentiation to find antiderivatives of simple functions, where no substitution is required	9, 1,0	1, 1, 0
	74	Solving initial value problems	4, 1,0	0, 1, 0
	75	Using sigma notation to write and evaluate a sum	5, 0,0	
	76	Demonstrating an understanding of the connection between the total change of a function and the area between its derivative and the input axis	2, 1,0	
	78	Finding the area of a plane region using limits	2, 1,0	
	79	Understanding the definition of a Riemann sum	5, 0,0	
	80	Evaluating a definite integral using limits	4, 1,0	
	81	Evaluating a definite integral using properties of definite integrals	5, 0,0	
	82	Evaluating a definite integral using the Fundamental Theorem of Calculus	5, 0,0	
	83	Understanding and using the Mean Value Theorem for Integrals	2, 1,0	

	85	Understanding and using the Second Fundamental Theorem of Calculus (i.e. the second half of the Fundamental Theorem of Calculus)	5, 0,0	
	86	Understanding and using the Net Change Theorem (Fundamental Theorem of Calculus Part II in applied context)	3, 2,0	
	87	Approximating definite integrals given the graph (and only the graph) of a function	1, 3,0	
	88	Using a change of variables (substitution) to find an indefinite integral	3, 0,0	
	89	Using the General Power Rule for Integration to find an indefinite integral	3, 0,0	
	90	Using a change of variables to evaluate a definite integral	3, 0,0	
	91	Evaluating a definite integral involving an even or odd function	0, 1,0	
	92	Interpreting in everyday language the meaning of a definite integral value in an applied context	1, 1,0	
	94	Approximating a definite integral using the Midpoint Rule	0, 2,0	
	95	Approximating a definite integral using the Trapezoidal Rule	0, 1,0	
	96	Approximating a definite integral using Simpson’s Rule	0, 1,0	
	97	Analyzing the approximate errors in the Trapezoidal Rule and Simpsonâ€™s Rule	0, 1,0	
	98	Using the Log Rule for Integration to integrate a rational function	1, 0,0	
	99	Integration of functions whose antiderivatives involve arcsin, arccos, and arctan	3, 0,0	1, 0, 0
	100	Integration of functions whose antiderivatives involve arccsc, arcsec, and arccot	0, 1,0	
	103	Developing properties of hyperbolic functions	0, 3,0	0, 1, 0
	104	Differentiation and integration of hyperbolic functions	0, 1,0	
	105	Developing properties of inverse hyperbolic functions	0, 1,0	
	106	Differentiation and integration of functions involving inverse hyperbolic functions	0, 1,0	
	159	Evaluation of an improper integral that has an infinite limit of integration	1, 0,0	
Both	72	Interpreting an indefinite integral as an antiderivative	1, 0,0	
	73	Using basic rules of antidifferentiation to find antiderivatives of simple functions, where no substitution is required	1, 0,0	
	74	Solving initial value problems	2, 0,0	
	79	Understanding the definition of a Riemann sum	1, 0,0	1, 0,0

	81	Evaluating a definite integral using properties of definite integrals	1, 0,0	1, 0,0
	82	Evaluating a definite integral using the Fundamental Theorem of Calculus	1, 0,0	1, 0,0
	85	Understanding and using the Second Fundamental Theorem of Calculus (i.e. the second half of the Fundamental Theorem of Calculus)	1, 0,0	1, 0,0
	86	Understanding and using the Net Change Theorem (Fundamental Theorem of Calculus Part II in applied context)	1, 0,0	1, 0,0
	88	Using a change of variables (substitution) to find an indefinite integral	2, 0,0	1, 0,0
	89	Using the General Power Rule for Integration to find an indefinite integral	2, 0,0	1, 0,0
	90	Using a change of variables to evaluate a definite integral	1, 0,0	
	92	Interpreting in everyday language the meaning of a definite integral value in an applied context	1, 0,0	1, 0,0
	98	Using the Log Rule for Integration to integrate a rational function	1, 0,0	
	102	Reviewing the basic integration rules involving elementary functions	1, 0,0	1, 0,0
	104	Differentiation and integration of hyperbolic functions	1, 2,0	1, 1,0
Calc II	72	Interpreting an indefinite integral as an antiderivative	13, 0,0	7, 0,0
	73	Using basic rules of antidifferentiation to find antiderivatives of simple functions, where no substitution is required	11, 0,0	6, 0,0
	74	Solving initial value problems	11, 4,0	4, 3,0
	75	Using sigma notation to write and evaluate a sum	15, 1,0	7, 0,0
	76	Demonstrating an understanding of the connection between the total change of a function and the area between its derivative and the input axis	16, 0,0	8, 0,0
	78	Finding the area of a plane region using limits	13, 2,0	7, 0,0
	79	Understanding the definition of a Riemann sum	16, 0,0	7, 0,0
	80	Evaluating a definite integral using limits	15, 2,0	6, 2,0
	81	Evaluating a definite integral using properties of definite integrals	16, 0,0	7, 0,0
	82	Evaluating a definite integral using the Fundamental Theorem of Calculus	16, 0,0	7, 0,0
	83	Understanding and using the Mean Value Theorem for Integrals	11, 5,0	3, 3,0
	85	Understanding and using the Second Fundamental Theorem of Calculus (i.e. the second half of the Fundamental Theorem of Calculus)	16, 0,0	7, 0,0

86	Understanding and using the Net Change Theorem (Fundamental Theorem of Calculus Part II in applied context)	13, 1,0	5, 1,0
87	Approximating definite integrals given the graph (and only the graph) of a function	9, 5,0	3, 3,0
88	Using a change of variables (substitution) to find an indefinite integral	17, 0,0	7, 0,0
89	Using the General Power Rule for Integration to find an indefinite integral	14, 1,0	5, 1,0
90	Using a change of variables to evaluate a definite integral	18, 0,0	8, 0,0
91	Evaluating a definite integral involving an even or odd function	10, 8,0	5, 2,0
92	Interpreting in everyday language the meaning of a definite integral value in an applied context	18, 0,0	7, 0,0
94	Approximating a definite integral using the Midpoint Rule	14, 4,0	6, 1,0
95	Approximating a definite integral using the Trapezoidal Rule	14, 6,0	6, 2,0
96	Approximating a definite integral using Simpson’s Rule	10, 5,1	4, 2,1
97	Analyzing the approximate errors in the Trapezoidal Rule and Simpsonâ€™s Rule	4, 14,0	2, 6,0
98	Using the Log Rule for Integration to integrate a rational function	14, 4,0	5, 2,0
99	Integration of functions whose antiderivatives involve arcsin, arccos, and arctan	17, 2,0	7, 0,0
100	Integration of functions whose antiderivatives involve arccsc, arcsec, and arccot	4, 11,1	0, 5,1
101	Using the method of completing the square to integrate a function	15, 6,0	4, 3,0
102	Reviewing the basic integration rules involving elementary functions	15, 5,0	4, 2,0
103	Developing properties of hyperbolic functions	3, 3,0	1, 2,0
104	Differentiation and integration of hyperbolic functions	1, 5,0	0, 2,0
105	Developing properties of inverse hyperbolic functions	1, 3,0	0, 1,0
106	Differentiation and integration of functions involving inverse hyperbolic functions	1, 2,0	0, 1,0
128	Describing integration as an accumulation process	12, 6,0	6, 0,0
145	Finding an antiderivative using integration by parts	22, 0,0	8, 0,0
146	Solving trigonometric integrals involving powers of sine and cosine	20, 1,0	7, 1,0
147	Solving trigonometric integrals involving powers of secant and tangent	16, 4,0	5, 2,0

148	Solving trigonometric integrals involving sine-cosine products with different angles	9, 7,0	2, 3,0
149	Using trigonometric substitution to solve an integral	20, 1,0	7, 1,0
151	Understanding the concept of partial fraction decomposition	22, 0,0	8, 0,0
152	Using partial fraction decomposition with linear factors to integrate rational functions	22, 0,0	8, 0,0
153	Using partial fraction decomposition with quadratic factors to integrate rational functions	18, 4,0	6, 2,0
154	Evaluation of indefinite integrals using a table of integrals	6, 6,0	1, 1,0
155	Evaluation of indefinite integrals using reduction formulas	4, 9,0	1, 2,0
156	Evaluation of indefinite integrals involving rational functions of sine and cosine	9, 8,0	1, 4,0
159	Evaluation of an improper integral that has an infinite limit of integration	20, 1,0	8, 0,0
160	Evaluation of an improper integral that has an infinite discontinuity	21, 1,0	8, 0,0

Applications of Integration

Course	No.	Question	All	B.Sc.
Calc I	77	Approximating the area of a plane region between the curve y=f(x) and x-axis	5, 0,0	
	84	Finding the average value of a function over a closed interval	3, 1,0	
	93	Finding position and velocity of a particle in rectilinear motion using integration	1, 2,0	
	127	Finding the area of a region between intersecting curves using integration	1, 0,0	1, 0, 0
	129	Finding the volume of a solid of revolution using the disk method	1, 0,0	
	130	Finding the volume of a solid of revolution using the washer method	1, 0,0	
	131	Finding the volume of a solid with known cross sections	1, 0,0	
	132	Finding the volume of a solid of revolution using the shell method	1, 0,0	
	133	Determining when to use the disk or shell method	1, 0,0	
	201	Finding the arc length of a curve given by a set of parametric equations	0, 1,0	0, 1, 0
	202	Finding the area of a surface of revolution (parametric form)	0, 1,0	0, 1, 0

Both	93	Finding position and velocity of a particle in rectilinear motion using integration	1, 1,0	
	126	Finding the area of a region between two curves using integration	1, 0,0	
Calc II	77	Approximating the area of a plane region between the curve y=f(x) and x-axis	15, 2,0	7, 1,0
	84	Finding the average value of a function over a closed interval	9, 8,0	3, 4,0
	93	Finding position and velocity of a particle in rectilinear motion using integration	14, 4,0	7, 1,0
	126	Finding the area of a region between two curves using integration	21, 0,0	8, 0,0
	127	Finding the area of a region between intersecting curves using integration	21, 0,0	7, 0,0
	129	Finding the volume of a solid of revolution using the disk method	18, 3,0	8, 0,0
	130	Finding the volume of a solid of revolution using the washer method	18, 3,0	8, 0,0
	131	Finding the volume of a solid with known cross sections	16, 5,0	7, 1,0
	132	Finding the volume of a solid of revolution using the shell method	15, 6,0	6, 2,0
	133	Determining when to use the disk or shell method	13, 8,0	5, 3,0
	134	Finding the arc length of a smooth curve	16, 4,0	5, 2,0
	135	Finding the area of a surface of revolution	9, 6,0	2, 2,0
	136	Finding the work done by a constant force	14, 3,1	6, 1,0
	137	Finding the work done by a variable force	15, 2,1	6, 1,0
	138	Understanding the definition of mass	5, 4,1	1, 3,0
	139	Finding the center of mass in a one-dimensional system	3, 7,1	1, 2,0
	140	Finding the center of mass in a two-dimensional system	4, 6,1	1, 2,0
	141	Finding the center of mass of a planar lamina	3, 3,1	1, 1,0
	142	Using the Theorem of Pappus to find the volume of a solid of revolution	0, 7,0	0, 3,0
	143	Finding fluid pressure and fluid force	3, 4,0	1, 1,0
	150	Using integrals to model and solve real-life applications	16, 4,0	5, 2,0
	185	Differentiation and integration of power series	15, 4,0	6, 1,0
	201	Finding the arc length of a curve given by a set of parametric equations	5, 6,0	1, 1,0
	202	Finding the area of a surface of revolution (parametric form)	2, 4,0	0, 1,0

	208	Finding the area of a region bounded by a polar graph	3, 5,0	2, 1,0
	210	Finding the arc length of a polar graph	1, 5,0	0, 2,0
	211	Finding the area of a surface of revolution (polar form)	0, 4,0	0, 2,0
	212	Finding the area of a surface of revolution (polar form)	0, 4,0	0, 2,0

Differential Equations

Course	No.		All	B.Sc.
Calc I	74	Solving initial value problems	4, 1,0	0, 1,0
	108	Checking if a function is a solutions to the given differential equation	1, 2,0	0, 1,0
	109	Using initial conditions to find particular solutions of differential equations	2, 0,0	1, 0,0
	110	Using slope fields to approximate solutions of differential equations	0, 1,0	0, 1,0
	111	Constructing a slope field for a given differential equations	0, 1,0	0, 1,0
	112	Using Euler's Method to approximate solutions of differential equations	0, 1,0	0, 1,0
	113	Using separation of variables to solve a simple differential equation	2, 0,0	
	114	Using exponential functions to model growth and decay in applied problems	2, 3,0	1, 2,0
	116	Recognizing and solving differential equations that can be solved by separation of variables	2, 0,0	
	118	Determining steady states and their stability	0, 1,0	0, 1,0
	120	Using logistic differential equations to model and solve applied problems	0, 1,0	0, 1,0
	121	Solving a first-order linear differential equation, and using linear differential equationsÂ to solve applied problems	0, 1,0	0, 1,0
	122	Analyzing predator-prey differential equations	0, 1,0	0, 1,0
	123	Analyzing competing-species differential equations	0, 1,0	0, 1,0
	124	Analyzing disease dynamics differential equations	0, 1,0	0, 1,0
Both	74	Solving initial value problems	2, 0,0	
	108	Checking if a function is a solutions to the given differential equation	3, 0,0	1, 0,0
	109	Using initial conditions to find particular solutions of differential equations	2, 0,0	1, 0,0
	114	Using exponential functions to model growth and decay in applied problems	1, 1,0	1, 1,0
	115	Using differential equations to model and solve applied problems	2, 0,0	1, 0,0
	117	Solving and analyzing logistic differential equations	0, 1,0	0, 1,0
Calc II	74	Solving initial value problems	11, 4,0	4, 3,0
	108	Checking if a function is a solutions to the given differential equation	15, 1,0	6, 0,0
	109	Using initial conditions to find particular solutions of differential equations	15, 3,0	4, 2,0
	110	Using slope fields to approximate solutions of differential equations	5, 8,0	0, 4,0
	111	Constructing a slope field for a given differential equations	4, 6,0	0, 3,0

	112	Using Euler's Method to approximate solutions of differential equations	3, 3,0	0, 1,0
	113	Using separation of variables to solve a simple differential equation	17, 2,0	6, 2,0
	114	Using exponential functions to model growth and decay in applied problems	10, 3,0	3, 0,0
	115	Using differential equations to model and solve applied problems	10, 8,0	3, 4,0
	116	Recognizing and solving differential equations that can be solved by separation of variables	15, 4,0	5, 3,0
	117	Solving and analyzing logistic differential equations	5, 8,0	1, 3,0
	118	Determining steady states and their stability	0, 5,0	0, 1,0
	119	Constructing phase diagrams	0, 1,0	
	120	Using logistic differential equations to model and solve applied problems	3, 7,0	0, 3,0
	121	Solving a first-order linear differential equation, and using linear differential equations to solve applied problems	5, 8,0	0, 3,0
	122	Analyzing predator-prey differential equations	2, 3,0	0, 1,0
	123	Analyzing competing-species differential equations	1, 3,0	
	124	Analyzing disease dynamics differential equations	0, 4,0	

Sequences and Series

Course	No.		All	B.Sc.
Calc II	162	Listing the terms of a sequence	18, 1,0	7, 0,0
	163	Writing a formula for the n-th term of a sequence	18, 3,0	5, 2,0
	164	Determining whether a sequence converges or diverges	21, 1,0	7, 1,0
	165	Using properties of monotonic sequences and bounded sequences to show convergence	14, 5,0	5, 1,0
	166	Understanding the definition of a convergent infinite series	19, 2,0	7, 1,0
	167	Knowing the convergence properties of geometric series	19, 2,0	6, 1,0
	168	Using the n-th Term Test for Divergence of an infinite series	19, 1,0	7, 0,0
	169	Using the Integral Test to determine whether an infinite series converges or diverges	14, 6,0	4, 2,0
	170	Knowing the convergence properties of p- and harmonic series	16, 4,0	5, 1,0
	171	Using the Direct Comparison Test to determine whether a series converges or diverges.	14, 7,0	5, 2,0
	172	Using the Limit Comparison Test to determine whether a series converges or diverges	13, 7,0	4, 2,0
	173	Using the Alternating Series Test to determine whether an infinite series converges	16, 4,0	5, 1,0
	174	Using the Alternating Series Remainder to approximate the sum of an alternating series	8, 9,0	3, 2,0
	175	Classification of convergent series as absolutely or conditionally convergent	12, 9,0	3, 4,0

	176	Using the Ratio Test to determine whether a series converges or diverges	21, 1,0	8, 0,0
	177	Using the Root Test to determine whether a series converges or diverges	13, 6,0	4, 2,0
	178	Applying the convergence and divergence tests for infinite series	18, 2,0	6, 1,0
	179	Finding polynomial approximations of elementary functions and compare them with the elementary functions	17, 3,1	6, 0,1
	180	Finding Taylor and Maclaurin polynomial approximations of elementary functions	21, 0,0	7, 0,0
	181	Using the remainder of a Taylor polynomial	10, 12,0	4, 4,0
	182	Understanding the definition of a power series	21, 1,0	7, 1,0
	183	Finding the radius and interval of convergence of a power series	21, 1,0	7, 1,0
	184	Determining the endpoint convergence of a power series	15, 5,0	5, 2,0
	185	Differentiation and integration of power series	15, 4,0	6, 1,0
	186	Finding the power series that represents a function	20, 2,0	7, 1,0
	187	Generate new power series from standard forms using algebra, substitution, differentiation, and/or integration	14, 7,0	5, 2,0
	188	Finding a Taylor or Maclaurin series for a function	22, 0,0	8, 0,0
	189	Finding a binomial series	5, 7,0	2, 2,0
	190	Using a basic list of Taylor series to find other Taylor series	12, 6,0	4, 2,0
Calc I and II	162	Listing the terms of a sequence	1, 0,0	
	179	Finding polynomial approximations of elementary functions and compare them with the elementary functions	1, 0,0	1, 0,0
	180	Finding Taylor and Maclaurin polynomial approximations of elementary functions	1, 0,0	1, 0,0

Curves

Course	No.		All	B.Sc.
Calc I	192	Understanding the definition of a conic section	0, 1,0	
	193	Analyzing and writing equations of parabolas using properties of parabolas	0, 1,0	
	194	Analyzing and writing equations of ellipses using properties of ellipses	0, 1,0	
	195	Analyzing and writing equations of hyperbolas using properties of hyperbolas	0, 1,0	
	196	Sketching the graph of a curve given by a set of parametric equations	7, 2,0	3, 1,0
	197	Eliminating the parameter in a set of parametric equations	4, 4,0	1, 3,0
	198	Finding a set of parametric equations to represent a curve	2, 6,0	0, 3,0

	200	Finding the slope of a tangent line to a curve given by a set of parametric equations	7, 2,0	3, 1,0
	201	Finding the arc length of a curve given by a set of parametric equations	0, 1,0	0, 1,0
	202	Finding the area of a surface of revolution (parametric form)	0, 1,0	0, 1,0
	203	Understanding the polar coordinate system	1, 2,0	1, 1,0
	204	Rewriting rectangular coordinates and equations in polar form and vice versa	1, 3,0	1, 1,0
	205	Sketching the graph of an equation given in polar form	2, 3,0	1, 1,0
	206	Finding the slope of a tangent line to a polar graph	1, 3,0	1, 1,0
	207	Identifying several types of special polar graphs	0, 4,0	0, 2,0
	209	Finding the points of intersection of two polar graphs	0, 2,0	0, 1,0
Calc I and II	203	Understanding the polar coordinate system	1, 0,0	
	204	Rewriting rectangular coordinates and equations in polar form and vice versa	1, 0,0	
	206	Finding the slope of a tangent line to a polar graph	1, 0,0	1, 0, 0
	209	Finding the points of intersection of two polar graphs	1, 1,0	
Calc II	192	Understanding the definition of a conic section	1, 2,0	
	193	Analyzing and writing equations of parabolas using properties of parabolas	1, 2,0	
	194	Analyzing and writing equations of ellipses using properties of ellipses	1, 2,0	
	195	Analyzing and writing equations of hyperbolas using properties of hyperbolas	1, 2,0	
	196	Sketching the graph of a curve given by a set of parametric equations	3, 1,0	
	197	Eliminating the parameter in a set of parametric equations	3, 2,0	
	198	Finding a set of parametric equations to represent a curve	3, 1,0	
	200	Finding the slope of a tangent line to a curve given by a set of parametric equations	3, 3,0	0, 1,0
	201	Finding the arc length of a curve given by a set of parametric equations	5, 6,0	1, 1,0
	202	Finding the area of a surface of revolution (parametric form)	2, 4,0	0, 1,0
	203	Understanding the polar coordinate system	2, 3,0	1, 0,0
	204	Rewriting rectangular coordinates and equations in polar form and vice versa	2, 2,0	1, 0,0
	205	Sketching the graph of an equation given in polar form	2, 2,0	1, 0,0
	206	Finding the slope of a tangent line to a polar graph	2, 2,0	
	207	Identifying several types of special polar graphs	0, 3,0	
	208	Finding the area of a region bounded by a polar graph	3, 5,0	2, 1,0
	209	Finding the points of intersection of two polar graphs	0, 3,0	0, 1,0
	210	Finding the arc length of a polar graph	1, 5,0	0, 2,0
	211	Finding the area of a surface of revolution (polar form)	0, 4,0	0, 2,0
	212	Finding the area of a surface of revolution (polar form)	0, 4,0	0, 2,0

213	Analyzing and writing polar equations of conics	0, 2,0